

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} \times & \times & 0 & 0 & 0 & 0 \\ 0 & \times & \times & 0 & 0 & 0 \\ 0 & 0 & \times & \times & 0 & 0 \\ 0 & 0 & 0 & \times & \times & 0 \\ 0 & 0 & 0 & 0 & \times & \times \\ 0 & 0 & 0 & 0 & 0 & \times \end{bmatrix}$$

$\Rightarrow$

quinddiagonal:

$$\begin{bmatrix} \times & \times & \times & 0 & 0 & 0 \\ 0 & \times & \times & \times & 0 & 0 \\ 0 & 0 & \times & \times & \times & 0 \\ 0 & 0 & 0 & \times & \times & \times \\ 0 & 0 & 0 & 0 & \times & \times \\ 0 & 0 & 0 & 0 & 0 & \times \end{bmatrix}$$

$\Rightarrow$

cyclic:

$$\begin{bmatrix} \times & \times & 0 & 0 & 0 & 0 \\ 0 & \times & \times & 0 & 0 & 0 \\ 0 & 0 & \times & \times & 0 & 0 \\ 0 & 0 & 0 & \times & \times & 0 \\ 0 & 0 & 0 & 0 & \times & \times \\ \times & 0 & 0 & 0 & 0 & \times \end{bmatrix}$$

$\Rightarrow$

WRITE TESTS LIKE A MATHEMATICIAN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

ISAACJ. LEE

$$\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ \times & \times & 0 & \times & 0 & \times \\ \times & 0 & \times & 0 & 0 & \times \\ 0 & \times & 0 & \times & 0 & \times \\ \times & \times & 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times & 0 & \times \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



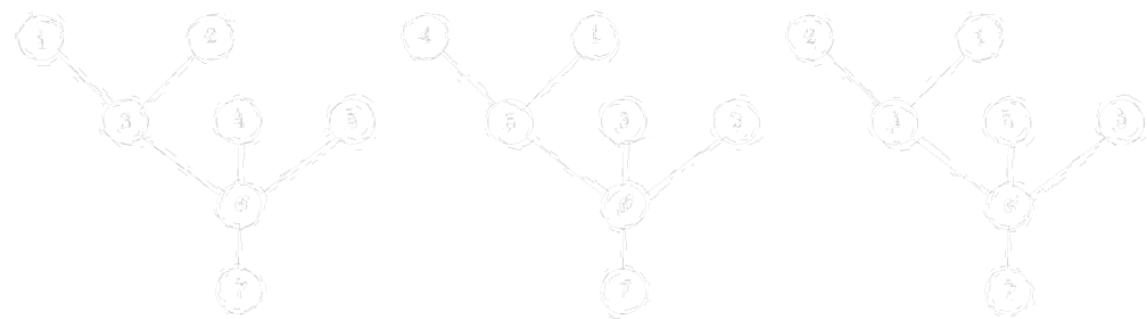
tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables. A *path* is an ordered set of edges $\{(i_k, j_k)\}_{k=1}^v \subseteq E$ joining the vertices α and β if $\alpha \in \{i_1, j_1\}$, $\beta \in \{i_v, j_v\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and arrowhead matrices correspond to trees, but this is not the case with either quinddiagonal or cyclic matrices when $n \geq 4$.

Given a tree G and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors (other words, we label the vertices from the top of the tree to the root). (As we have already said it, relabelling a graph is equivalent to permuting the rows and the columns of the and the matrix.)

Very often the path between two vertices is unique and in general, such an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

1

5 Rules of Writing Tests

Filter:

Module:

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Running:

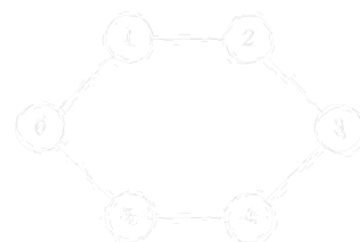
acyclic:

$$\begin{bmatrix} \times & \circ & \times & \circ & \times & \circ \\ \times & \times & \times & \times & \times & \times \\ \times & \circ & \circ & \circ & \circ & \times \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} \times & \times & \circ & \circ & \circ & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} \times & \circ & \times & \circ & \times & \circ \\ \circ & \times & \circ & \times & \times & \circ \\ \times & \circ & \times & \circ & \circ & \times \\ \circ & \times & \circ & \times & \circ & \times \\ \times & \times & \circ & \circ & \times & \times \\ \circ & \circ & \times & \times & \circ & \times \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree can be monotonically ordered and, in general, such an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

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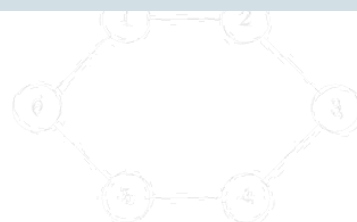
1 assertion of 1 passed, 0 failed.

If, if, if

cyclic:



$\Rightarrow$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

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2 assertions of 2 passed, 0 failed.

If, if, if

Use common, everyday words

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} \times & \emptyset & \times & \emptyset & \times & \emptyset \\ \emptyset & \times & \emptyset & \times & \times & \emptyset \\ \times & \emptyset & \times & \emptyset & \emptyset & \times \\ \emptyset & \times & \emptyset & \times & \emptyset & \times \\ \times & \times & \emptyset & \emptyset & \times & \emptyset \\ \emptyset & \emptyset & \times & \times & \emptyset & \times \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{kj} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity

5 Rules of Writing Tests

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3 assertions of 3 passed, 0 failed.

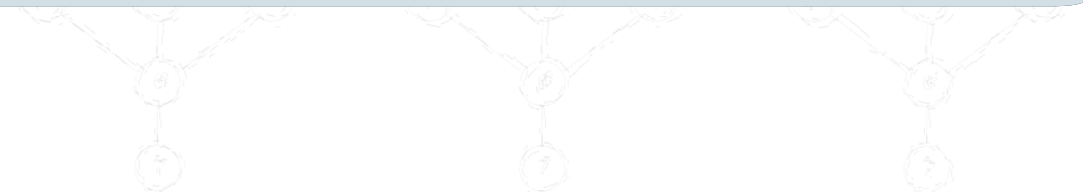
If, if, if

Use common, everyday words

Write less with theorems and new terms

$$\begin{bmatrix} \times & \circ & \times & \circ & \times & \circ \\ \circ & \times & \circ & \times & \times & \circ \\ \times & \circ & \times & \circ & \circ & \times \\ \circ & \times & \circ & \times & \circ & \times \\ \times & \times & \circ & \circ & \times & \circ \\ \circ & \circ & \times & \times & \circ & \times \end{bmatrix}$$

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5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



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4 assertions of 4 passed, 0 failed.

If, if, if

Use common, everyday words

Write less with theorems and new terms

All your basis are belong to us

$$\begin{pmatrix} \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \end{pmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

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5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline ▼

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5 assertions of 5 passed, 0 failed.

If, if, if

Use common, everyday words

Write less with theorems and new terms

All your basis are belong to us

1 picture = 1000 words

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the four matrices we have just displayed, but its graph,



tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_k, j_k)\}_{k=1}^v \subseteq E$ is called a *path* joining the vertices α and β if $\alpha \in \{i_1, j_1\}$, $\beta \in \{i_v, j_v\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and symmetric matrices correspond to trees, but this is not the case with either quintadiagonal or cyclic matrices when $p \geq 4$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (T, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

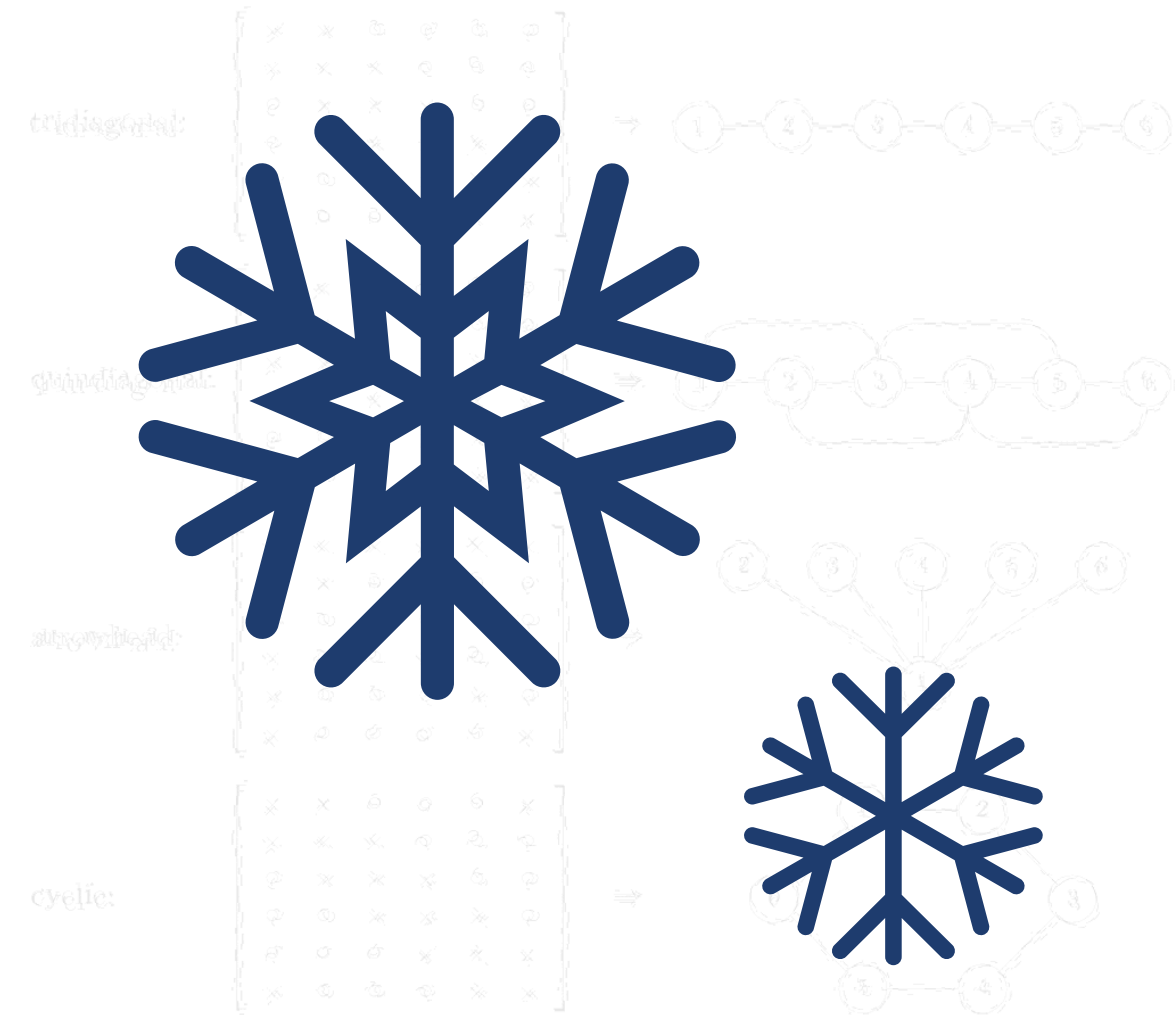
Every rooted tree can be monotonically ordered and, in general, such an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

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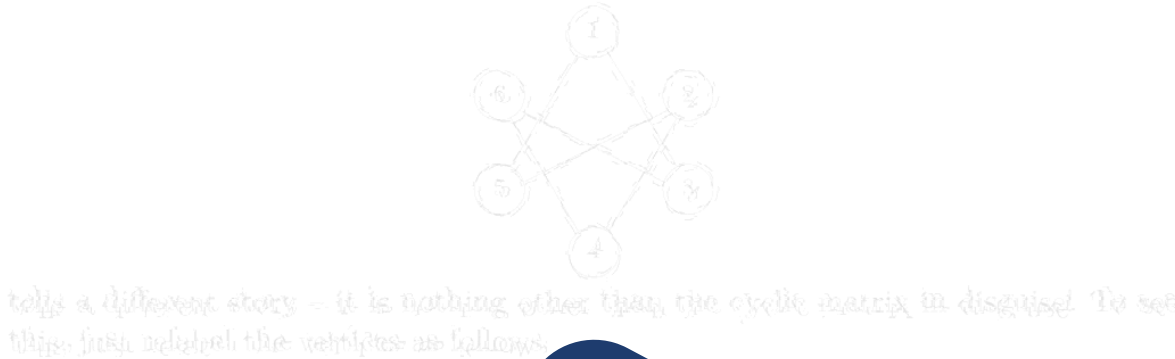


The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

IF

$$\begin{bmatrix} \times & 0 & \times & \times & 0 \\ 0 & \times & 0 & \times & 0 \\ \times & 0 & \times & 0 & \times \\ 0 & \times & 0 & \times & 0 \\ \times & \times & 0 & 0 & \times \\ 0 & 0 & \times & \times & 0 \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 4, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 2.$$

This, of course, is equivalent to reordering the equations and variables.

An ordered set of edges $\{(i_k, j_k)\}_{k=1}^p$ is called a *simple path* if the vertices i_k and j_k satisfy $i_k \in \{i_{k-1}, j_{k-1}\}$, $j_k \in \{i_k, j_k\}$ and $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. A simple path does not visit any vertex more than once. We say that G is a *simple path* if each two vertices of V are joined by a unique simple path. Both the 5×5 banded and the 6×6 cyclic matrices correspond to trees, but this is not the case with the 6×6 symmetric or cyclic matrices when $p \geq 4$.

Given a tree $T = (G, r)$ and an arbitrary vertex $r \in V$, the pair (T, r) is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T has a natural partial ordering, which can be obtained by an analogy with a family tree. Thus, the root r is the *predecessor* of all vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for α itself, as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabeling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree can be monotonically ordered and, in general, an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



THEN

Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

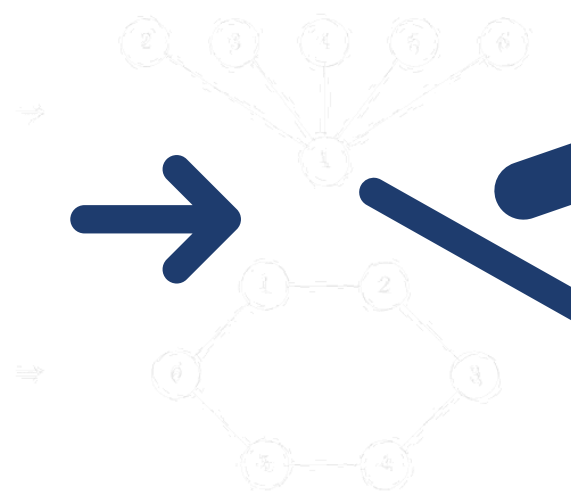
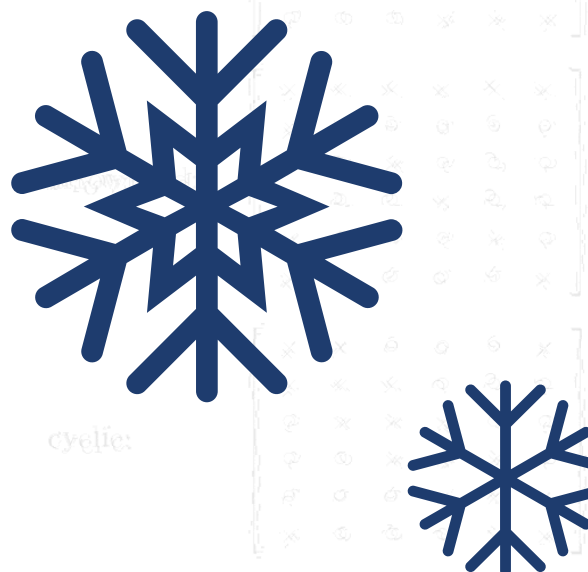
tridiagonal:

$$\begin{bmatrix} \times & \times & 0 & 0 & 0 & 0 \\ 0 & \times & \times & 0 & 0 & 0 \\ 0 & 0 & \times & \times & \times & 0 \\ 0 & 0 & 0 & \times & \times & \times \\ 0 & 0 & 0 & 0 & \times & \times \end{bmatrix}$$



quintidiagonal:

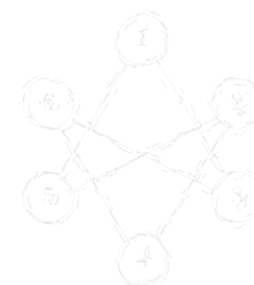
$$\begin{bmatrix} \times & \times & \times & 0 & 0 & 0 \\ \times & \times & \times & \times & 0 & 0 \\ 0 & \times & \times & \times & \times & 0 \\ 0 & 0 & \times & \times & \times & \times \\ 0 & 0 & 0 & \times & \times & \times \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & \times & 0 \\ \times & 0 & \times & 0 & 0 & \times \\ 0 & \times & 0 & \times & 0 & \times \\ \times & \times & 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times & 0 & \times \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

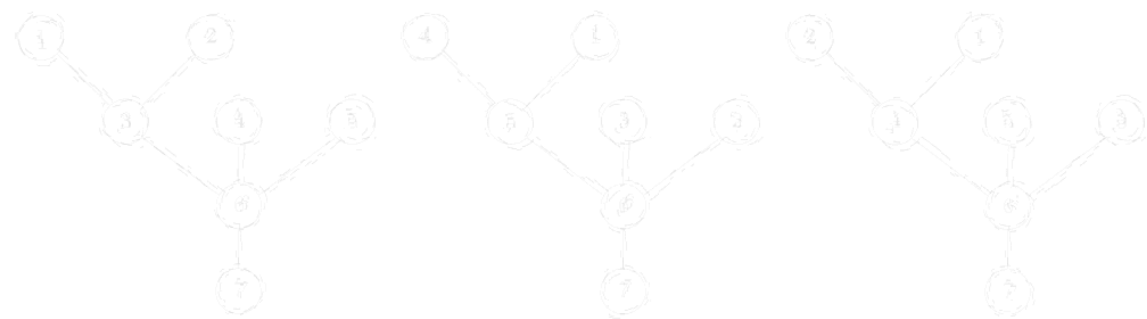
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_k, j_k)\}_{k=1}^v \subseteq E$ is called a *path* joining the vertices i and j if $i \in \{i_k, j_k\}$, $j \in \{i_v, j_v\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and arrowhead matrices correspond to trees, but this is not the case with either quintidiagonal or cyclic matrices.

Let T be a tree T and an arbitrary vertex $r \in V$ the pair $T = (G, r)$ is called a *rooted tree*. We say that r is the *root*. Unlike an ordinary graph, the partial order $\prec$ on V can best be explained by an analogy with a family tree. The root r is the predecessor of all the vertices in V . If p is the predecessor of r , then p is the predecessor of all the vertices in V that are descendants of r . Moreover, every $q \in V$ is joined to r by a unique simple path. We say that the rooted tree T is *monotonically ordered* if every vertex i is labeled before all its predecessors; in other words, if the label the vertices i is less than the label of the root. (As we have already said, relabelling a graph is equivalent to reordering the rows and the columns of the underlying matrix.)

The graph of a tree can be monotonically ordered and, in general, such an ordering is not unique. We now give three monotonic orderings of the tree T in Fig. 11.1.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

```
assert.speaker().getsPersonal();
```

```
await sing('Happy Birthday');
```

```
assert.audience().isHappy();
```

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} x & x & 0 & 0 & 0 & 0 \\ 0 & x & x & x & 0 & 0 \\ 0 & 0 & x & x & x & 0 \\ 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & 0 & 0 & x \end{bmatrix}$$

quindagonal:

$$\begin{bmatrix} x & x & x & 0 & 0 & 0 \\ x & x & x & x & 0 & 0 \\ 0 & x & x & x & x & 0 \\ 0 & 0 & x & x & x & x \\ 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & x & x \end{bmatrix}$$

symmetric:

$$\begin{bmatrix} x & x & x & x & x & x \\ x & x & 0 & 0 & 0 & 0 \\ x & 0 & x & x & x & x \\ x & 0 & 0 & x & x & 0 \\ x & 0 & 0 & 0 & x & 0 \\ x & 0 & 0 & 0 & 0 & x \end{bmatrix}$$

cyclic:

$$\begin{bmatrix} x & x & 0 & 0 & 0 & x \\ x & x & x & 0 & 0 & 0 \\ 0 & x & x & x & 0 & 0 \\ 0 & 0 & x & x & x & 0 \\ 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & x & x \end{bmatrix}$$

SUSPECT 1

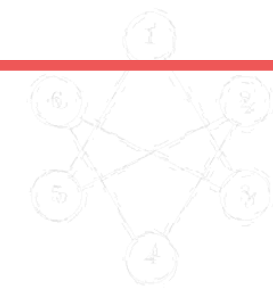
OBSERVER /

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

COMPUTED

$$\begin{bmatrix} 0 & x & 0 & 0 & 0 & x \\ x & x & x & 0 & 0 & 0 \\ 0 & x & 0 & x & 0 & x \\ x & x & 0 & 0 & x & 0 \\ 0 & 0 & x & x & 0 & x \\ x & 0 & x & x & 0 & x \end{bmatrix}$$

It is clear, at once, that we are studying the kind of the stars and the cycle matrices which are known just displayed, but its graph,



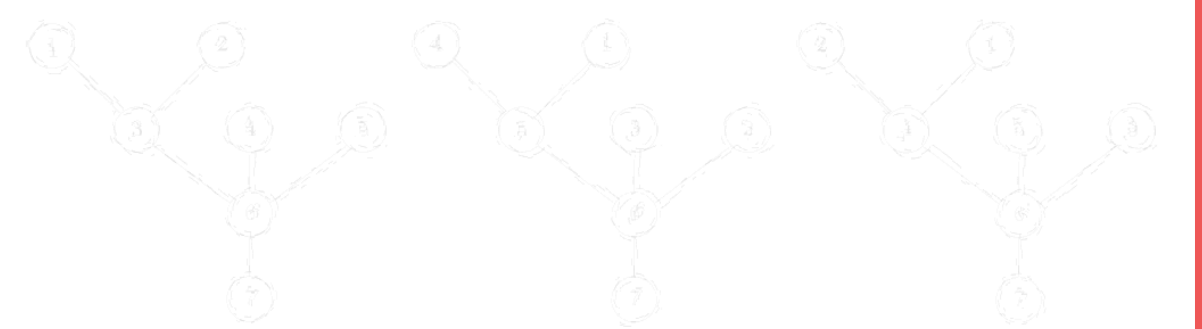
tells a different story - it is nothing other than the cycle matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

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$$l_{k,j} = \frac{a_{kj}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

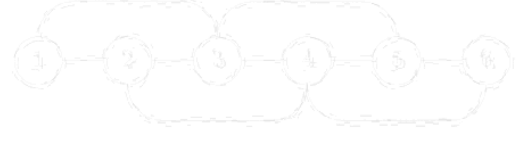
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

SUSPECT 2

tridiagonal:

$$\begin{bmatrix} x & x & 0 & 0 & 0 & 0 \\ 0 & x & x & x & 0 & 0 \\ 0 & 0 & x & x & x & 0 \\ 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & 0 & 0 & x \end{bmatrix}$$


quindagonal:

$$\begin{bmatrix} x & x & x & 0 & 0 & 0 \\ x & x & x & x & 0 & 0 \\ 0 & x & x & x & x & 0 \\ 0 & 0 & x & x & x & x \\ 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & x & x \end{bmatrix}$$


symmetric:

$$\begin{bmatrix} x & x & x & x & x & x \\ x & x & 0 & 0 & 0 & 0 \\ x & 0 & x & x & x & x \\ x & 0 & 0 & x & x & 0 \\ x & 0 & 0 & 0 & x & 0 \\ x & 0 & 0 & 0 & 0 & x \end{bmatrix}$$


cyclic:

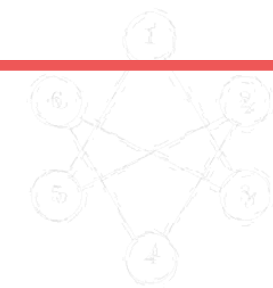
$$\begin{bmatrix} x & x & 0 & 0 & 0 & x \\ x & x & x & 0 & 0 & 0 \\ 0 & x & x & x & 0 & 0 \\ 0 & 0 & x & x & x & 0 \\ 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & x & x \end{bmatrix}$$


The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

EMBER DATA / FORM BUILDER

$$\begin{bmatrix} 0 & x & 0 & 0 & 0 & x \\ x & x & 0 & x & 0 & x \\ 0 & x & 0 & x & 0 & x \\ x & x & 0 & 0 & x & 0 \\ 0 & 0 & x & x & 0 & x \\ 0 & 0 & 0 & x & x & x \end{bmatrix}$$

It is clear, at once, that it is equivalent to a kind of a star graph, with the same structure as the one shown just displayed, but its graph,



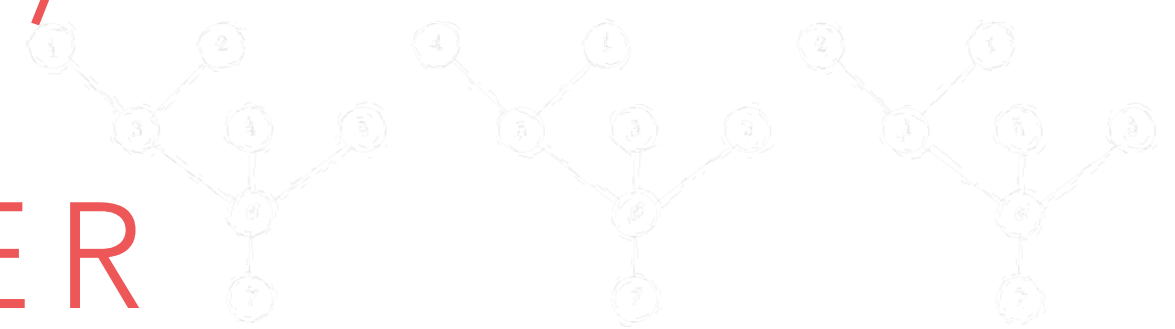
tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables. An ordered set of edges $\{(i_k, j_k)\}_{k=1}^v \subseteq E$ is called a *path* joining the vertices α and β if $\alpha \in \{i_1, j_1\}$, $\beta \in \{i_v, j_v\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and symmetric matrices correspond to trees, but this is not the case with either quindagonal or cyclic matrices when $p \geq 4$.

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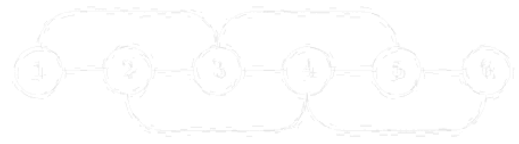


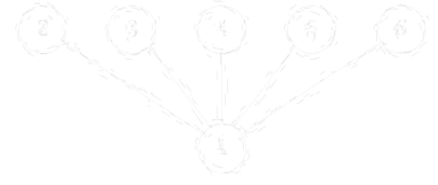
Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

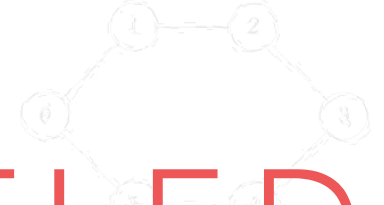
$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal: $\begin{bmatrix} x & x & 0 & 0 & 0 & 0 \\ 0 & x & x & x & 0 & 0 \\ 0 & 0 & x & x & x & 0 \\ 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & 0 & 0 & x \end{bmatrix} \Rightarrow$ 

quindagonal: $\begin{bmatrix} x & x & x & 0 & 0 & 0 \\ x & x & x & x & 0 & 0 \\ 0 & x & x & x & x & 0 \\ 0 & 0 & x & x & x & x \\ 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & x & x \end{bmatrix} \Rightarrow$ 

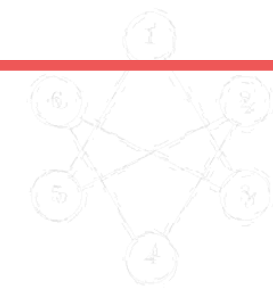
symmetric: $\begin{bmatrix} x & x & x & x & x & x \\ x & x & 0 & 0 & 0 & 0 \\ x & 0 & x & 0 & 0 & 0 \\ x & 0 & 0 & x & 0 & 0 \\ x & 0 & 0 & 0 & x & 0 \\ x & 0 & 0 & 0 & 0 & x \end{bmatrix} \Rightarrow$ 

cyclic: $\begin{bmatrix} x & x & 0 & 0 & 0 & x \\ x & x & x & 0 & 0 & 0 \\ 0 & x & x & x & 0 & 0 \\ 0 & 0 & x & x & x & 0 \\ 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & x & x \end{bmatrix} \Rightarrow$ 

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

STATE $\begin{bmatrix} x & x & 0 & x & 0 \\ 0 & x & x & x & 0 \\ 0 & x & x & 0 & x \\ x & x & 0 & 0 & x & 0 \\ 0 & 0 & x & x & 0 & x \end{bmatrix}$

It is clear, at once, that we are studying a kind of a star, and this does not mean it is just displayed, but its graph,



tells a different story - it is nothing other than the cycle matrix in disguise. To see this, just relabel the vertices as follows:

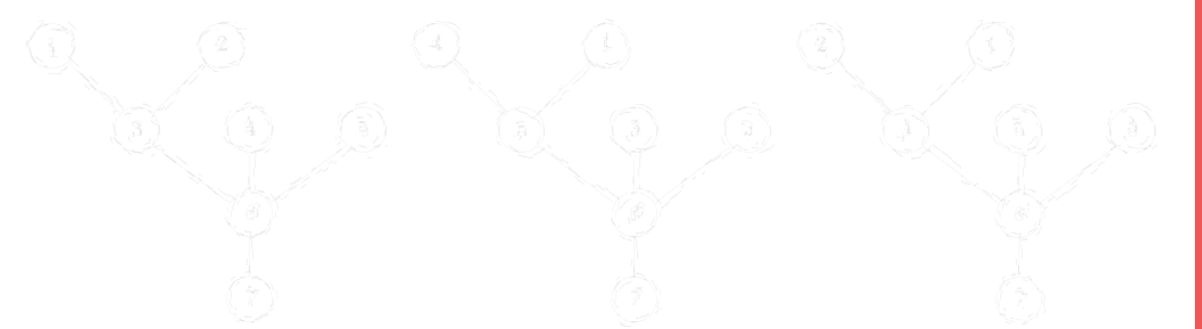
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

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Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} x & x & 0 & 0 & 0 & 0 \\ 0 & x & x & x & 0 & 0 \\ 0 & 0 & x & x & x & 0 \\ 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & 0 & 0 & x \end{bmatrix}$$

quintadiagonal:

$$\begin{bmatrix} x & x & x & 0 & 0 & 0 \\ x & x & x & x & 0 & 0 \\ 0 & x & x & x & x & 0 \\ 0 & 0 & x & x & x & x \\ 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & x & x \end{bmatrix}$$

symmetric:

$$\begin{bmatrix} x & x & x & x & x & x \\ x & x & 0 & 0 & 0 & 0 \\ x & 0 & x & 0 & 0 & 0 \\ x & 0 & 0 & x & 0 & 0 \\ x & 0 & 0 & 0 & x & 0 \\ x & 0 & 0 & 0 & 0 & x \end{bmatrix}$$

cyclic:

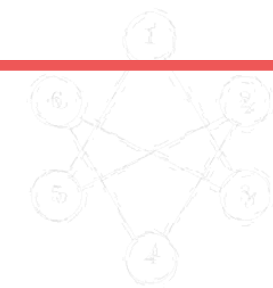
$$\begin{bmatrix} x & x & 0 & 0 & 0 & x \\ x & x & x & 0 & 0 & 0 \\ 0 & x & x & x & 0 & 0 \\ 0 & 0 & x & x & x & 0 \\ 0 & 0 & 0 & x & x & x \\ x & 0 & 0 & 0 & x & x \end{bmatrix}$$

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

LEAKAGE

$$\begin{bmatrix} x & x & 0 & 0 & 0 & 0 & 0 \\ 0 & x & x & x & 0 & 0 & 0 \\ 0 & 0 & x & x & x & 0 & 0 \\ 0 & 0 & 0 & x & x & x & 0 \\ 0 & 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & 0 & 0 & 0 & x \end{bmatrix}$$

It is clear, at once, that we are studying the kind of leakage of the down-converter when we know just displayed, but its graph,



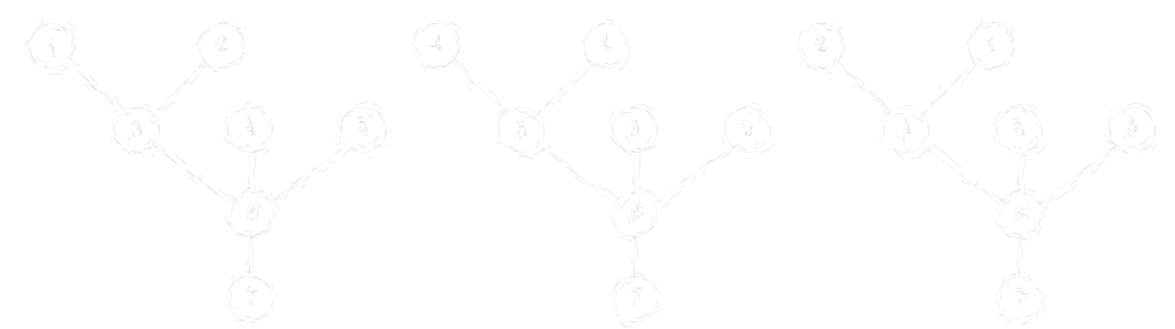
tells a different story - it is nothing other than the cycle matrix in disguise. To see this, just relabel the vertices as follows:

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$$\begin{bmatrix} x & x & 0 & 0 & 0 & 0 \\ 0 & x & x & x & 0 & 0 \\ 0 & 0 & x & x & x & 0 \\ 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & 0 & 0 & x \end{bmatrix}$$

quindagonal:

$$\begin{bmatrix} x & x & x & 0 & 0 & 0 \\ x & x & x & x & 0 & 0 \\ x & x & x & x & x & 0 \\ 0 & x & x & x & x & x \\ 0 & 0 & x & x & x & x \\ 0 & 0 & 0 & x & x & x \end{bmatrix}$$

symmetric:

$$\begin{bmatrix} x & x & x & x & x & x \\ x & x & 0 & 0 & 0 & 0 \\ x & 0 & x & x & x & x \\ x & 0 & 0 & x & x & 0 \\ x & 0 & 0 & 0 & x & 0 \\ x & 0 & 0 & 0 & 0 & x \end{bmatrix}$$

cyclic:

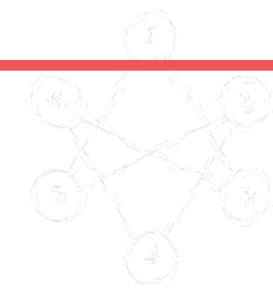
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The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

PRIVILEGE

$$\begin{bmatrix} x & 0 & x & 0 & 0 & x & 0 \\ 0 & x & 0 & x & 0 & 0 & 0 \\ x & 0 & x & 0 & x & 0 & x \\ 0 & x & 0 & x & 0 & 0 & 0 \\ x & 0 & x & 0 & 0 & x & 0 \\ 0 & 0 & 0 & x & 0 & x & x \\ 0 & 0 & x & 0 & x & 0 & x \end{bmatrix}$$

It is clear, at once, that we are studying a kind of hierarchy of the form mentioned above, not just displayed, but its graph,



tells a different story - it is nothing other than the cycle matrix in disguise. To see this, just relabel the vertices as follows:

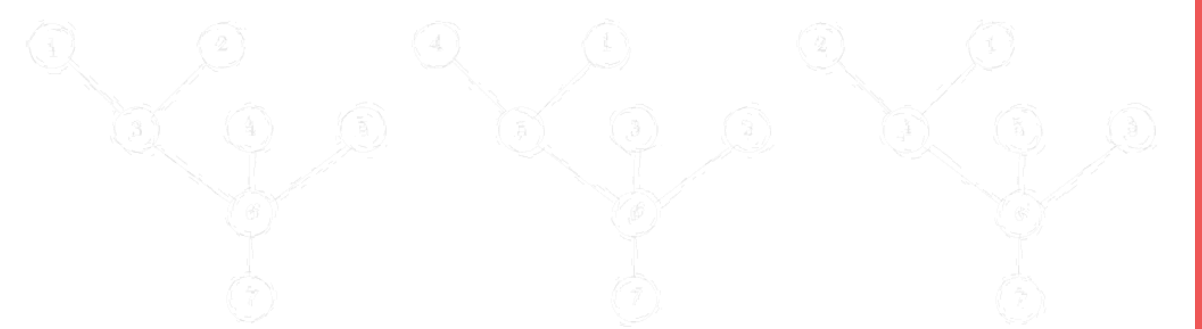
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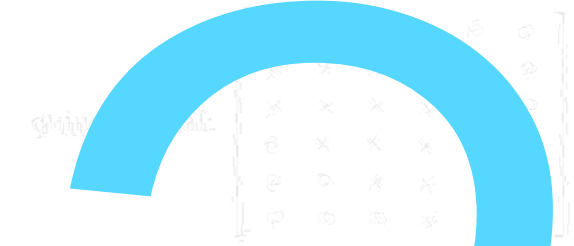
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Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the four matrices we have seen.

USE

COMMON

EVERYDAY

WORDS



tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

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Theorem 1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

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triagonal:
$$\begin{bmatrix} x & x & 0 & 0 & 0 & 0 \\ x & x & x & 0 & 0 & 0 \\ 0 & x & x & x & 0 & 0 \\ 0 & 0 & x & x & x & 0 \\ 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & x & x \end{bmatrix}$$

quindagonal:
$$\begin{bmatrix} x & x & x & 0 & 0 & 0 \\ x & x & x & x & 0 & 0 \\ 0 & x & x & x & x & 0 \\ 0 & 0 & x & x & x & x \\ 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & x & x \end{bmatrix}$$

symmetric:
$$\begin{bmatrix} x & x & x & x & x & x \\ x & x & 0 & 0 & 0 & 0 \\ x & 0 & x & 0 & 0 & 0 \\ x & 0 & 0 & x & 0 & 0 \\ x & 0 & 0 & 0 & x & 0 \\ x & 0 & 0 & 0 & 0 & x \end{bmatrix}$$

cyclic:
$$\begin{bmatrix} x & x & 0 & 0 & 0 & x \\ x & x & x & 0 & 0 & 0 \\ 0 & x & x & x & 0 & 0 \\ 0 & 0 & x & x & x & 0 \\ 0 & 0 & 0 & x & x & x \\ x & 0 & 0 & 0 & x & x \end{bmatrix}$$

tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

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This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_k, j_k)\}_{k=1}^v \subseteq E$ is called a *path* joining the vertices α and β if $\alpha \in \{i_1, j_1\}$, $\beta \in \{i_v, j_v\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both triagonal and symmetric matrices correspond to trees, but this is not the case with either quindagonal or cyclic matrices when $p \geq 4$.

Given a tree G and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree can be monotonically ordered and, in general, still an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} x & x & x & 0 & 0 & x & 0 \\ x & x & 0 & 0 & 0 & x & 0 \\ 0 & x & x & x & 0 & 0 & x \\ 0 & 0 & x & x & x & 0 & 0 \\ 0 & 0 & 0 & x & x & x & 0 \\ x & x & 0 & 0 & x & 0 & x \\ 0 & 0 & x & x & 0 & x & x \end{bmatrix}$$

As a friend of mine, who is studying for a first or second year of the degree programme, once told me, just displayed, but its graph,

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal: $\begin{bmatrix} \times & \times & & & & \\ & \times & \times & & & \\ & & \times & \times & & \\ \times & & & \times & \times & \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ \circ \\ 2 \\ \circ \\ 3 \\ \circ \\ 4 \\ \circ \\ 5 \\ \circ \\ 6 \end{array}$

quintadiagonal: $\begin{bmatrix} \times & \times & \times & & & \\ & \times & \times & \times & & \\ & & \times & \times & \times & \\ \times & & & \times & \times & \times \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ \circ \\ 2 \\ \circ \\ 3 \\ \circ \\ 4 \\ \circ \\ 5 \\ \circ \\ 6 \end{array}$

symmetric: $\begin{bmatrix} \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \end{bmatrix} \Rightarrow \begin{array}{c} 2 \\ \circ \\ 3 \\ \circ \\ 4 \\ \circ \\ 5 \\ \circ \\ 6 \end{array}$

cyclic: $\begin{bmatrix} \times & \times & & & & \times \\ \times & \times & \times & & & \\ & \times & \times & \times & & \\ & & \times & \times & \times & \\ & & & \times & \times & \times \\ & & & & \times & \times \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ \circ \\ 2 \\ \circ \\ 3 \\ \circ \\ 4 \\ \circ \\ 5 \end{array}$

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

FAMILIAR TO MANY $\begin{bmatrix} \times & & \times & & & \\ & \times & & \times & & \\ \times & & \times & & \times & \\ & \times & & \times & & \times \\ & & \times & & \times & \\ \times & & \times & & \times & \end{bmatrix}$

As a friend of mine, who is studying for his Ph.D. in the theory of the discrete continuous, once saw this, just displayed, but its graph,



tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

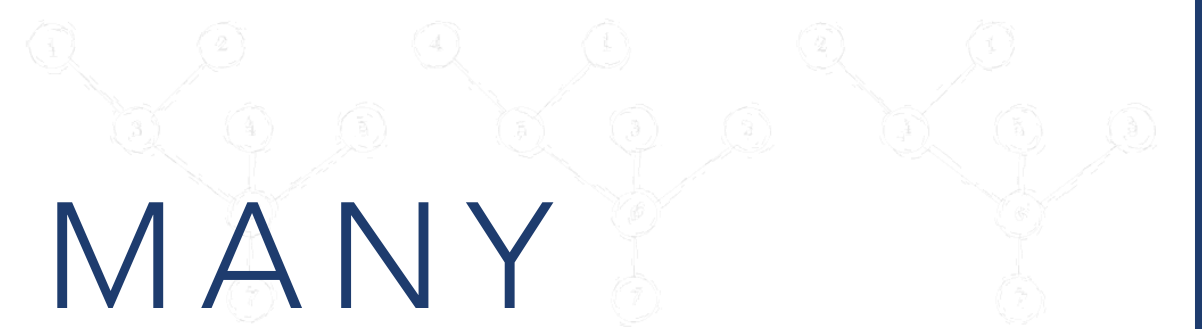
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_k, j_k)\}_{k=1}^v \subseteq E$ is called a *path* joining the vertices α and β if $\alpha \in \{i_1, j_1\}$, $\beta \in \{i_v, j_v\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and symmetric matrices correspond to trees, but this is not the case with either quintadiagonal or cyclic matrices when $p \geq 4$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (T, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree can be monotonically ordered and, in general, such an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{kj} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$


```
assert.dom('[data-test-message]')  
  .hasText(  
    'Thanks for signing up!',  
    'The user sees a welcome message.'  
  );
```

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} \times & \times & & & & \\ & \times & \times & & & \\ & & \times & \times & & \\ & & & \times & \times & \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix}$$


quintadiagonal:

$$\begin{bmatrix} \times & & & & & \\ & \times & & & & \\ & & \times & & & \\ & & & \times & & \\ & & & & \times & \\ & & & & & \times \end{bmatrix}$$


symmetric:

$$\begin{bmatrix} \times & \times & & & & \\ & \times & \times & & & \\ & & \times & \times & & \\ & & & \times & \times & \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix}$$


cyclic:

$$\begin{bmatrix} \times & & & & \times \\ & \times & & & \\ & & \times & & \\ & & & \times & \\ & & & & \times \end{bmatrix}$$


THEOREM 11.1. IF f IS CONTINUOUS IN $[a, b]$, AND IF $f(a)f(b) < 0$, THEN f MUST HAVE A ZERO IN (a, b) .

The graph of a matrix often reveals at a glance its structure, which might not be evident from its sparsity pattern. This can be seen in the matrix

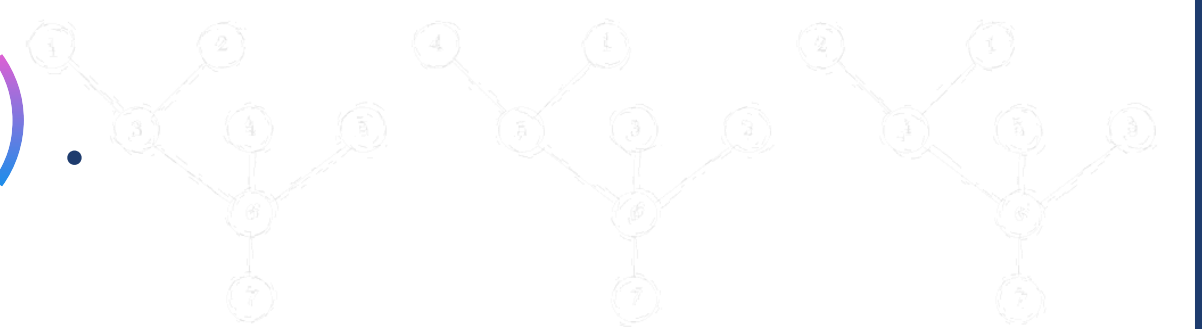
$$\begin{bmatrix} \times & & \times & & \times & & \\ & \times & & \times & & \times & \\ \times & & \times & & \times & & \\ & \times & & \times & & \times & \\ \times & & \times & & \times & & \\ & \times & & \times & & \times & \\ \times & & \times & & \times & & \end{bmatrix}$$

is a kind of cross. When it is translated to a kind of the graph, the structure is more just displayed, but its graph,



tells a different story - it is nothing other than the cycle matrix in disguise. To see this, just relabel the vertices as follows:

$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 3, 6 \rightarrow 6$.
 This of course, is equivalent to reordering (simultaneously) the equations and variables. A graph with a set of edges $\{(i_j, j_k)\} \subseteq E$ is called a *path* joining the vertices i and j if $i \in \{i_1, i_2\}$, $j \in \{i_v, j_v\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and arrowhead matrices correspond to trees, but this is not the case with either quintadiagonal or cyclic matrices when $p \geq 4$.
 A rooted tree $T = (V, E)$ and a vertex $r \in V$, the pair $T = (V, E)$ is called a *rooted tree*, which is due to the root. Unlike an arbitrary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors (in other words, we label the vertices from the top of the tree to the bottom). We have already said that labelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.
 Every rooted tree can be monotonically ordered and, in general, such an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{kj} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story - it is nothing other than the cycle matrix in disguise. To see this, first relabel the vertices as follows:



Dashboard

Explore

Settings



vertices more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and arrowhead matrices correspond to trees, but this is not the case with either quindagonal or cyclic matrices when $p \geq 4$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (T, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the

'[data-test-link="Dashboard"]'

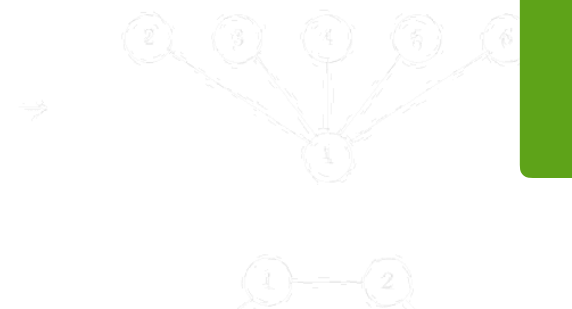
'[data-test-link="Explore"]'

'[data-test-link="Settings"]'

just displayed, but its graph,

$$t_{k,j} = \frac{a_{k,j}}{t_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

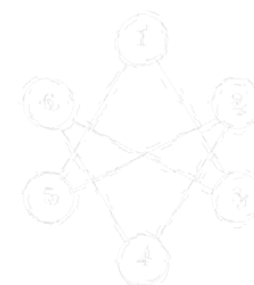
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



Save

Cancel

+



tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 5, 2 \rightarrow 2, 3 \rightarrow 4, 4 \rightarrow 6, 5 \rightarrow 3, 6 \rightarrow 1.$$

After reordering (simultaneously) the equations and variables, the matrix becomes tridiagonal. A sequence $\{(i_k, j_k)\}_{k=1}^n \subseteq E$ is called a *path* joining the vertices i_1 and j_n if for every $k = 1, 2, \dots, n-1$ the set $\{(i_k, j_k) \cap \{(i_{k+1}, j_{k+1})\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and arrowhead matrices correspond to trees.

This is not the case with either quintadiagonal or cyclic matrices when $p \geq 4$. Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (T, r)$ is called a *rooted tree* while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the

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'[data-test-button="Cancel"]'

'[data-test-button="Add item"]'

just displayed, but its graph,

$$t_{k,j} = \frac{a_{k,j}}{t_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a list examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:



Name*

quintidiagonal:



Description

symmetric:



cyclic:



permuting the rows and the columns of the underlying matrix.)

Every rooted tree can be monotonically ordered and, in general, such an ordering is not unique. We now give three monotonic orderings of the same rooted tree.

matrix in disguise. To see

$6 \rightarrow 3$.

equations and variables.
 with joining the vertices v
 $v = 1$ the set $\{j_k, j_k\} \cap$
 if it does not visit any
 members of V are joined by
 lines corresponding to nodes.
 matrices when $p \geq 4$.
 $= (G, \tau)$ is called a rooted
 admits a natural partial
 a family tree. Thus, the
 se vertices are successors
 the path and we designate
 son of α and a successor
 if each vertex is labelled
 times from the top of the
 graph is isomorphic to

'[data-test-field="Name"]'

'[data-test-field="Description"]'

just displayed, but its graph,

$$t_{k,j} = \frac{a_{k,j}}{t_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a list examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:



Name

Little Bobby Tables

quintidiagonal:



Description

Better not drop me!

strongly cyclic:

cyclic:



permuting the rows and the columns of the underlying matrix.)

Every rooted tree can be monotonically ordered and, in general, such an ordering is not unique. We now give three monotonic orderings of the same rooted tree.

matrix in disguise. To see

$6 \rightarrow 3$.

equations and variables. with joining the vertices v $v = 1$ the set $\{j_k, j_\ell\} \cap$ h if it does not visit any h members of V are joined by h lines corresponding to nodes. h matrices when $p \geq 4$. $h = (E, \ell)$ is called a rooted h admits a natural partial h a family tree. Thus, the h vertices are h the path and we designate h son of α and a successor h if each vertex is labelled h from the top of the h graph is isomorphic to

'[data-test-field="Name"]'

'[data-test-field="Description"]'

just displayed, but its graph,

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} \times & \times & \circ & \circ & \circ & \circ \\ \circ & \times & \times & \times & \circ & \circ \\ \circ & \circ & \times & \times & \times & \times \\ \circ & \circ & \circ & \times & \times & \times \\ \circ & \circ & \circ & \circ & \times & \times \\ \circ & \circ & \circ & \circ & \circ & \times \end{bmatrix}$$

quintadiagonal:

$$\begin{bmatrix} \times & \times & \times & \times & \times & \circ \\ \circ & \times & \times & \times & \times & \circ \\ \circ & \circ & \times & \times & \times & \circ \\ \circ & \circ & \circ & \times & \times & \times \\ \circ & \circ & \circ & \circ & \times & \times \\ \circ & \circ & \circ & \circ & \circ & \times \end{bmatrix}$$

symmetric:

$$\begin{bmatrix} \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \end{bmatrix}$$

cyclic:

$$\begin{bmatrix} \times & \times & \circ & \circ & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \circ & \circ & \times & \times & \times & \times \\ \circ & \circ & \circ & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \end{bmatrix}$$

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} \times & \circ & \times & \circ & \times \\ \circ & \times & \circ & \times & \times \\ \times & \circ & \times & \circ & \circ \\ \circ & \times & \circ & \times & \circ \\ \times & \times & \circ & \circ & \times \\ \circ & \circ & \times & \times & \circ \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices we have just displayed, but its graph,

WRITE LESS

WITH

THEOREMS

AND

NEW TERMS

tells a different story - it is nothing other than the cyclic matrix displayed. To see this, first relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables. An ordered set of edges $\{(i_k, j_k)\}_{k=1}^p \subseteq E$ is called a *path* joining the vertices α and β if $\alpha = i_1, j_1 = i_2, j_2 = i_3, j_3 = i_4, j_4 = \dots = i_p, j_p = \beta$ and for every $k = 1, 2, \dots, p-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and symmetric matrices correspond to trees, but this is not the case with either quintadiagonal or cyclic matrices when $p \geq 4$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (T, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is *topologically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree can be topologically ordered and, in general, still an ordering is not unique. We now give three successive orderings of the same rooted tree.

Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is topologically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} \times & \times & & & & \\ & \times & \times & & & \\ & & \times & \times & & \\ & & & \times & \times & \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix}$$


quintidiagonal:

$$\begin{bmatrix} \times & \times & \times & \times & \times & \\ & \times & \times & \times & \times & \\ & & \times & \times & \times & \\ & & & \times & \times & \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix}$$


symplectic:

$$\begin{bmatrix} \times & & & \times & & \\ & \times & & & \times & \\ & & \times & & & \times \\ & & & \times & & \\ & & & & \times & \\ & & & & & \times \end{bmatrix}$$


cyclic:

$$\begin{bmatrix} \times & \times & & & \times & \\ & \times & \times & & & \\ & & \times & \times & & \\ & & & \times & \times & \\ & & & & \times & \times \\ & \times & & & & \times \end{bmatrix}$$


THE GRAPH OF A MATRIX OFTEN REVEALS AT A GLANCE ITS STRUCTURE, WHICH MIGHT NOT BE EVIDENT FROM ITS SPARSITY PATTERN. THIS CAN BE SEEN IN THE MATRIX

$$\begin{bmatrix} \times & & \times & & \times & & \\ & \times & & \times & & \times & \\ \times & & \times & & \times & & \\ & \times & & \times & & \times & \\ \times & & \times & & \times & & \\ & \times & & \times & & \times & \\ \times & & \times & & \times & & \end{bmatrix}$$

It is clear that such a matrix is symmetric (as indicated by the symmetry of the sparsity pattern) and we know just displayed, but its graph,

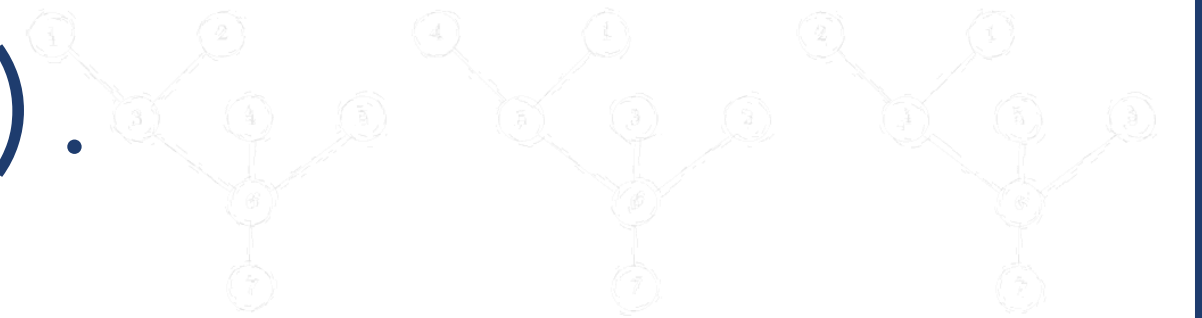


tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$1 \rightarrow 4, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 3, 6 \rightarrow 3.$

Of course, it is equally so concerning simultaneously the equations and variables. A graph with a set of edges $\{(i_j, j_k)\} \subseteq E$ is called a *path* joining the vertices i and j if $i \in \{i_1, i_2\}$, $j \in \{j_v, j_v\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and arrowhead matrices correspond to trees, but this is not the case with either quintidiagonal or cyclic matrices when $p \geq 4$. A tree $T = (G, r)$ is called a *rooted tree* if $r \in V$, the node r is called a *root* (this is due to the motivation). Unlike a simple path, there is a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors (in other words, we label the vertices from the top of the tree to the root, i.e. we label exactly said, labelling a graph is tantamount to permutation of rows and the columns of the underlying matrix.)

Every rooted tree can be monotonically ordered and, in general, such an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{kj} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal: $\begin{bmatrix} \times & \times & 0 & 0 & 0 & 0 \\ 0 & \times & \times & 0 & 0 & 0 \\ 0 & 0 & \times & \times & 0 & 0 \\ 0 & 0 & 0 & \times & \times & 0 \\ 0 & 0 & 0 & 0 & \times & \times \\ 0 & 0 & 0 & 0 & 0 & \times \end{bmatrix} \Rightarrow$

quintadiagonal: $\begin{bmatrix} \times & \times & \times & 0 & 0 & 0 \\ \times & \times & \times & \times & 0 & 0 \\ \times & \times & \times & \times & \times & 0 \\ 0 & \times & \times & \times & \times & \times \\ 0 & 0 & \times & \times & \times & \times \\ 0 & 0 & 0 & \times & \times & \times \end{bmatrix} \Rightarrow$

symmetric: $\begin{bmatrix} \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \end{bmatrix} \Rightarrow$

cyclic: $\begin{bmatrix} \times & \times & 0 & 0 & 0 & \times \\ \times & \times & \times & \times & 0 & 0 \\ 0 & \times & \times & \times & \times & 0 \\ 0 & 0 & \times & \times & \times & \times \\ 0 & 0 & 0 & \times & \times & \times \\ \times & 0 & 0 & 0 & \times & \times \end{bmatrix} \Rightarrow$

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & \times & 0 \\ \times & 0 & \times & 0 & 0 & \times \\ 0 & \times & 0 & \times & 0 & \times \\ \times & \times & 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times & 0 & \times \end{bmatrix}$$

It is clear, at once, that we are dealing with a kind of a star, and the above matrix is what you know just displayed, but its graph,



tells a different story - it is nothing other than the cycle matrix in disguise. To see this, just relabel the vertices as follows:

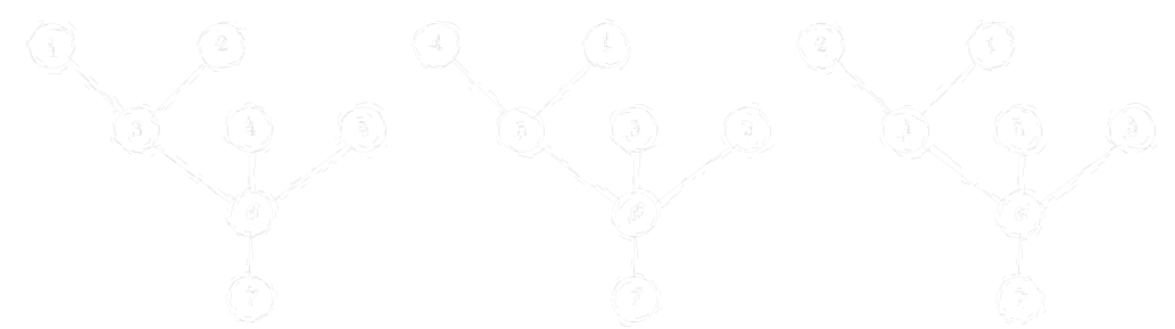
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_k, j_k)\}_{k=1}^n \subseteq E$ is called a *path* joining the vertices α and β if $\alpha \in \{i_1, j_1\}$, $\beta \in \{i_n, j_n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if any two members of V are joined by a unique simple path. Given undirected and acyclic graphs correspond to trees, but this is not the case with either quintadiagonal or cyclic matrices when $n \geq 4$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (T, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, for $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree can be monotonically ordered and, in general, still an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} \times & \times & 0 & 0 & 0 & 0 \\ 0 & \times & \times & 0 & 0 & 0 \\ 0 & 0 & \times & \times & 0 & 0 \\ 0 & 0 & 0 & \times & \times & 0 \\ 0 & 0 & 0 & 0 & \times & \times \\ 0 & 0 & 0 & 0 & 0 & \times \end{bmatrix}$$

quintadiagonal:

$$\begin{bmatrix} \times & \times & \times & 0 & 0 & 0 \\ 0 & \times & \times & \times & 0 & 0 \\ 0 & 0 & \times & \times & \times & 0 \\ 0 & 0 & 0 & \times & \times & \times \\ 0 & 0 & 0 & 0 & \times & \times \\ 0 & 0 & 0 & 0 & 0 & \times \end{bmatrix}$$

symmetric:

$$\begin{bmatrix} \times & \times & \times & \times & \times & \times \\ \times & \times & 0 & 0 & 0 & 0 \\ \times & 0 & \times & \times & \times & \times \\ \times & 0 & 0 & 0 & 0 & 0 \\ \times & 0 & 0 & 0 & 0 & 0 \\ \times & 0 & 0 & 0 & 0 & \times \end{bmatrix}$$

cyclic:

$$\begin{bmatrix} \times & \times & 0 & 0 & 0 & \times \\ \times & \times & \times & 0 & 0 & 0 \\ 0 & \times & \times & \times & 0 & 0 \\ 0 & 0 & \times & \times & \times & 0 \\ 0 & 0 & 0 & \times & \times & \times \\ \times & 0 & 0 & 0 & \times & \times \end{bmatrix}$$

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & \times & 0 \\ \times & 0 & \times & 0 & 0 & \times \\ 0 & \times & 0 & \times & 0 & \times \\ \times & \times & 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times & 0 & \times \end{bmatrix}$$

As is clear, it is not obvious at a glance at the shape of the above matrix, which was merely just displayed, but its graph,



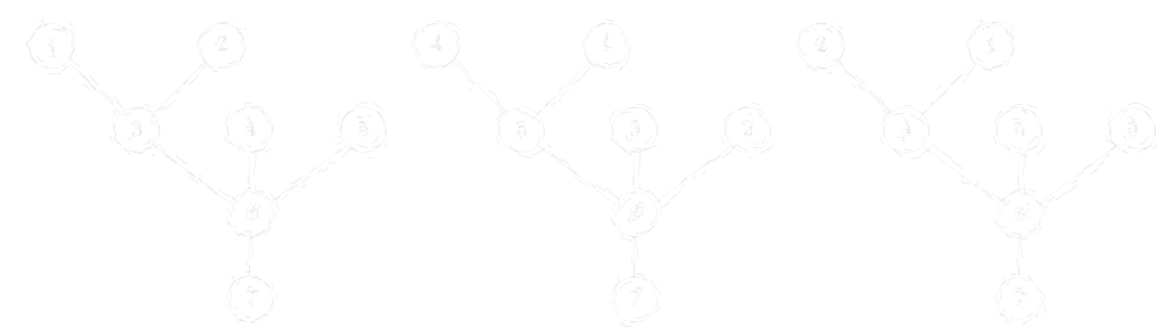
tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables. An ordered set of edges $\{(i_k, j_k)\}_{k=1}^v \subseteq E$ is called a *path* joining the vertices α and β if $\alpha \in \{i_1, j_1\}$, $\beta \in \{i_v, j_v\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and symmetric matrices correspond to trees, but this is not the case with either quintadiagonal or cyclic matrices when $n \geq 4$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (T, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree can be monotonically ordered and, in general, still an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$


```
import { fillIn } from '@ember/test-helpers';

export async function fillForm(fields) {
  for (const { label, value } of fields) {
    // input, textarea
    await fillIn(`[data-test-field="${label}"]`, value);
  }
};
```

```
import { fillForm } from '../helpers/my-test-helpers';

...

test('User can create account', async function(assert) {

  await visit('/signup');

  await fillForm([

    { label: 'Name', value: 'Little Bobby Tables' },

    { label: 'Email', value: 'little.bobby@gmail.com' }

  ]);

  ...

});
```

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

NEW TERM

tridiagonal:

$$\begin{bmatrix} \times & \times & & & & \\ & \times & \times & & & \\ & & \times & \times & & \\ & & & \times & \times & \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix} \Rightarrow \begin{array}{c} \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \end{array}$$

quintadiagonal:

$$\begin{bmatrix} \times & \times & \times & & & \\ & \times & \times & \times & & \\ & & \times & \times & \times & \\ & & & \times & \times & \times \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix} \Rightarrow \begin{array}{c} \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \end{array}$$

symmetric:

$$\begin{bmatrix} \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \end{bmatrix} \Rightarrow \begin{array}{c} \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \end{array}$$

cyclic:

$$\begin{bmatrix} \times & \times & & & & \times \\ \times & \times & \times & & & \\ & \times & \times & \times & & \\ & & \times & \times & \times & \\ & & & \times & \times & \times \\ & & & & \times & \times \end{bmatrix} \Rightarrow \begin{array}{c} \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \end{array}$$

UBIQUITOUS

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

IDEA

$$\begin{bmatrix} \times & & \times & & \times & & \\ & \times & & \times & & \times & \\ & & \times & & \times & & \\ & & & \times & & \times & \\ & & & & \times & & \times \\ & & & & & \times & \\ & & & & & & \times \end{bmatrix}$$

It is clear, at once, that we are dealing with a kind of the stars of the famous mathematicians who are known just displayed, but its graph,



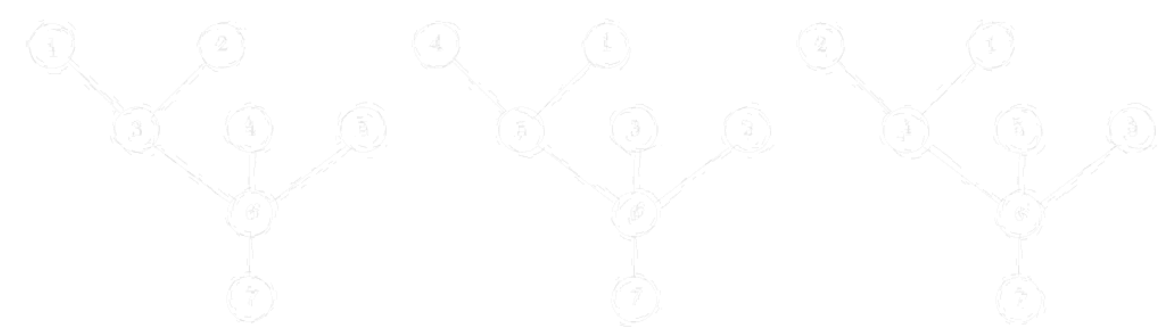
tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables. An ordered set of edges $\{(i_k, j_k)\}_{k=1}^v \subseteq E$ is called a *path* joining the vertices α and β if $\alpha \in \{i_1, j_1\}$, $\beta \in \{i_v, j_v\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and symmetric matrices correspond to trees, but this is not the case with either quintadiagonal or cyclic matrices when $p \geq 4$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (T, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree can be monotonically ordered and, in general, such an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity

patterns) and their graphs.

tridiagonal:

$$\begin{bmatrix} \times & \times & & & & \\ & \times & \times & & & \\ & & \times & \times & & \\ & & & \times & \times & \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix}$$



quintadiagonal:

$$\begin{bmatrix} \times & & & & & \\ & \times & & & & \\ & & \times & & & \\ & & & \times & & \\ & & & & \times & \\ & & & & & \times \end{bmatrix}$$



symplectic:

$$\begin{bmatrix} \times & & & & & \\ & \times & & & & \\ & & \times & & & \\ & & & \times & & \\ & & & & \times & \\ & & & & & \times \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} \times & & & & & \\ & \times & & & & \\ & & \times & & & \\ & & & \times & & \\ & & & & \times & \\ & & & & & \times \end{bmatrix}$$



tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 4, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 1.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

A graph with vertices $\{1, 2, \dots, n\}$ is called a *path* if there exists a sequence of vertices $v_1, v_2, \dots, v_n$ such that v_i and v_{i+1} are adjacent for $i = 1, 2, \dots, n-1$. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and symplectic matrices correspond to trees, but this is not the case with either quintadiagonal or cyclic matrices when $n \geq 4$.

A rooted tree $T = (V, E)$ is called a *rooted tree* if there exists a vertex $r \in V$ such that r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a *predecessor* of v and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors (in other words, if we label the vertices from the top of the tree to the bottom, we label them in increasing order). Labelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.

Every rooted tree can be monotonically ordered and, in general, such an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{kj} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

IF f IS CONTINUOUS IN

$[a, b]$, AND IF $f(a)f(b) < 0$,

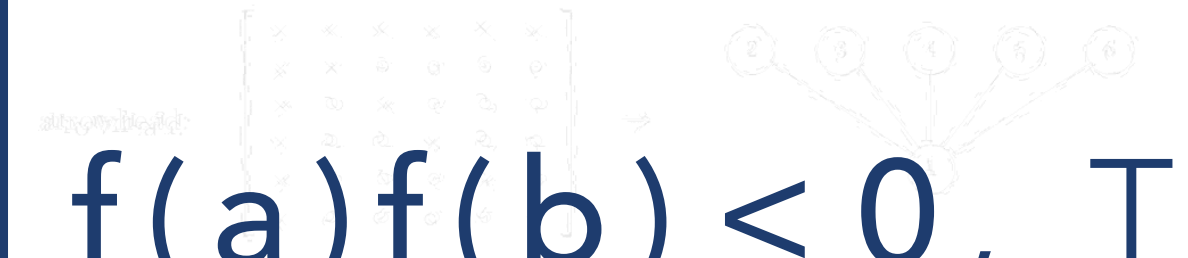
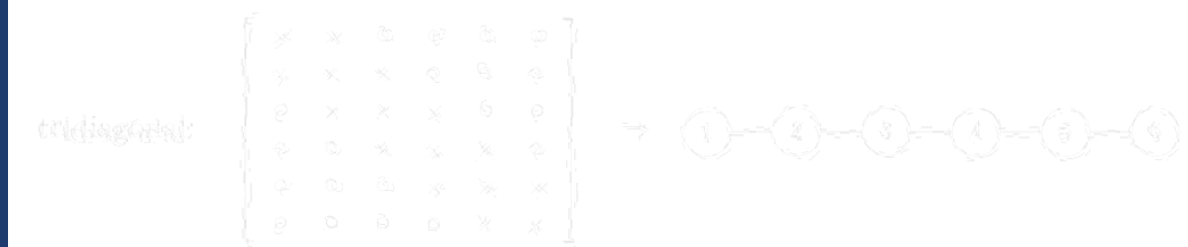
THEN f MUST HAVE A

ZERO IN (a, b) .

$$\begin{bmatrix} \times & \times & & & & \\ & \times & \times & & & \\ & & \times & \times & & \\ & & & \times & \times & \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix}$$

is a good idea, when is studying the limit of the sum of the terms, not only the terms are known just displayed, but its graph,

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & \times & 0 \\ \times & 0 & \times & 0 & 0 & \times \\ 0 & \times & 0 & \times & 0 & \times \\ \times & \times & 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times & 0 & \times \end{bmatrix}$$

As a friend of mine, who is studying for a first or second year of the degree in mathematics, once told me, just displayed, but its graph,



tells a different story - it is nothing other than the cycle matrix in disguise. To see this, just relabel the vertices as follows:

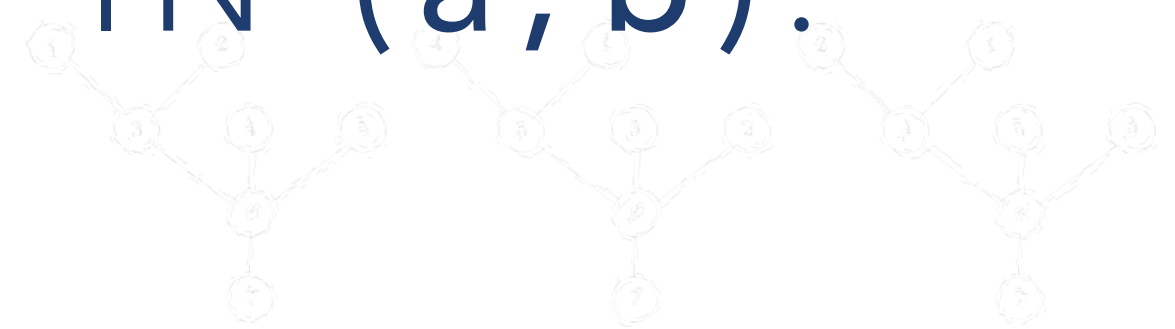
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_k, j_k)\}_{k=1}^n \subseteq E$ is called a *path* joining the vertices α and β if $\alpha \in \{i_1, j_1\}$, $\beta \in \{i_n, j_n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex (other than α and β) more than once. We say that G has a *line* if each two members of V are joined by a unique simple path. Both tridiagonal and symmetric matrices correspond to lines, but this is not the case with either quintadiagonal or cyclic matrices when $n \geq 4$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (T, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a unique path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree can be monotonically ordered and, in general, still an ordering is not fixed. We now give three canonical orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{kj} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

```
hooks.beforeEach(function(assert) {  
  ...  
  // Example: assert.isEnabled('Submit', 'Woot!');  
  assert.isEnabled = (label, message) => {  
    assert.dom(`[data-test-button="${label}"]`)  
      .doesNotHaveAttribute('disabled', message);  
  };  
  ...  
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} \times & \times & \circ & \circ & \circ & \circ \\ \circ & \times & \times & \times & \circ & \circ \\ \circ & \circ & \times & \times & \times & \times \\ \circ & \circ & \circ & \times & \times & \times \\ \circ & \circ & \circ & \circ & \times & \times \end{bmatrix}$$

quintadiagonal:

$$\begin{bmatrix} \times & \times & \circ & \circ & \circ & \circ \\ \times & \times & \times & \times & \circ & \circ \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \end{bmatrix}$$

symmetric:

$$\begin{bmatrix} \times & \times & \circ & \circ & \circ & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \end{bmatrix}$$

cyclic:

$$\begin{bmatrix} \circ & \circ & \times & \times & \circ & \circ \\ \circ & \circ & \circ & \times & \times & \times \\ \times & \circ & \circ & \times & \times & \times \\ \times & \circ & \circ & \times & \times & \times \\ \times & \circ & \circ & \times & \times & \times \\ \times & \circ & \circ & \times & \times & \times \end{bmatrix}$$

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} \times & \circ & \times & \circ & \times \\ \circ & \times & \circ & \times & \times \\ \times & \circ & \times & \circ & \circ \\ \circ & \times & \circ & \times & \circ \\ \times & \times & \circ & \circ & \times \\ \times & \circ & \times & \times & \times \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices we have just displayed, but its graph,

ALL YOUR BASIS ARE BELONG TO US

tells a different story – it is nothing other than the matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables. An ordered set of edges $\{(i_k, j_k)\}_{k=1}^v \subseteq E$ is called a *path* joining the vertices i_1 and j_v if $i_k \neq i_l$, $j_k \neq j_l$ and for every $k = 1, 2, \dots, v-1$ the set $\{(i_k, j_k) \cap (i_{k+1}, j_{k+1})\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and arrowhead matrices correspond to trees, but this is not the case with either quintadiagonal or cyclic matrices when $p \geq 4$.

Given a tree G and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree can be monotonically ordered and, in general, still an ordering is a story. We give three monotonic orderings of the same rooted tree.

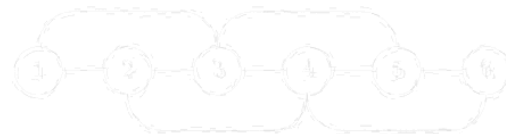


Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

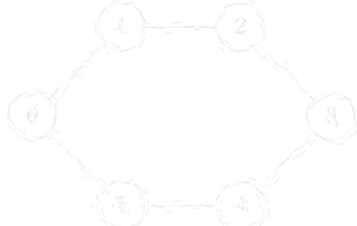
$$l_{kj} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal: $\begin{bmatrix} \times & \times & & & & \\ & \times & \times & & & \\ & & \times & \times & & \\ & & & \times & \times & \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix} \Rightarrow$ 

quindagonal: $\begin{bmatrix} \times & \times & \times & & & \\ & \times & \times & \times & & \\ & & \times & \times & \times & \\ & & & \times & \times & \times \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix} \Rightarrow$ 

symmetric: $\begin{bmatrix} \times & \times & \times & \times & \times & \times \\ & \times & \times & \times & \times & \times \\ & & \times & \times & \times & \times \\ & & & \times & \times & \times \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix} \Rightarrow$ 

cyclic: $\begin{bmatrix} \times & \times & & & & \times \\ & \times & \times & & & \\ & & \times & \times & & \\ & & & \times & \times & \\ & & & & \times & \times \\ & \times & & & & \times \end{bmatrix} \Rightarrow$ 

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$\begin{bmatrix} \times & \times & \times & \times & \times & \times & \times \\ & \times & \times & \times & \times & \times & \times \\ & & \times & \times & \times & \times & \times \\ & & & \times & \times & \times & \times \\ & & & & \times & \times & \times \\ & & & & & \times & \times \\ & & & & & & \times \end{bmatrix}$

As a final, of course, question is whether it is likely to be the graph of the dense symmetric matrix now shown just displayed, but its graph,



tells a different story - it is nothing other than the cycle matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_k, j_k)\}_{k=1}^v \subseteq E$ is called a *path* joining the vertices α and β if $\alpha \in \{i_1, j_1\}$, $\beta \in \{i_v, j_v\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and arrowhead matrices correspond to trees, but this is not the case with either quindagonal or cyclic matrices when $p \geq 4$.

Given a tree G and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all its vertices in $V \setminus \{r\}$ and all vertices are successors of r . Moreover, every vertex $v \in V \setminus \{r\}$ is joined to r by a simple path, and we designate each vertex along this path, except for r and v , as a *predecessor* of v and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

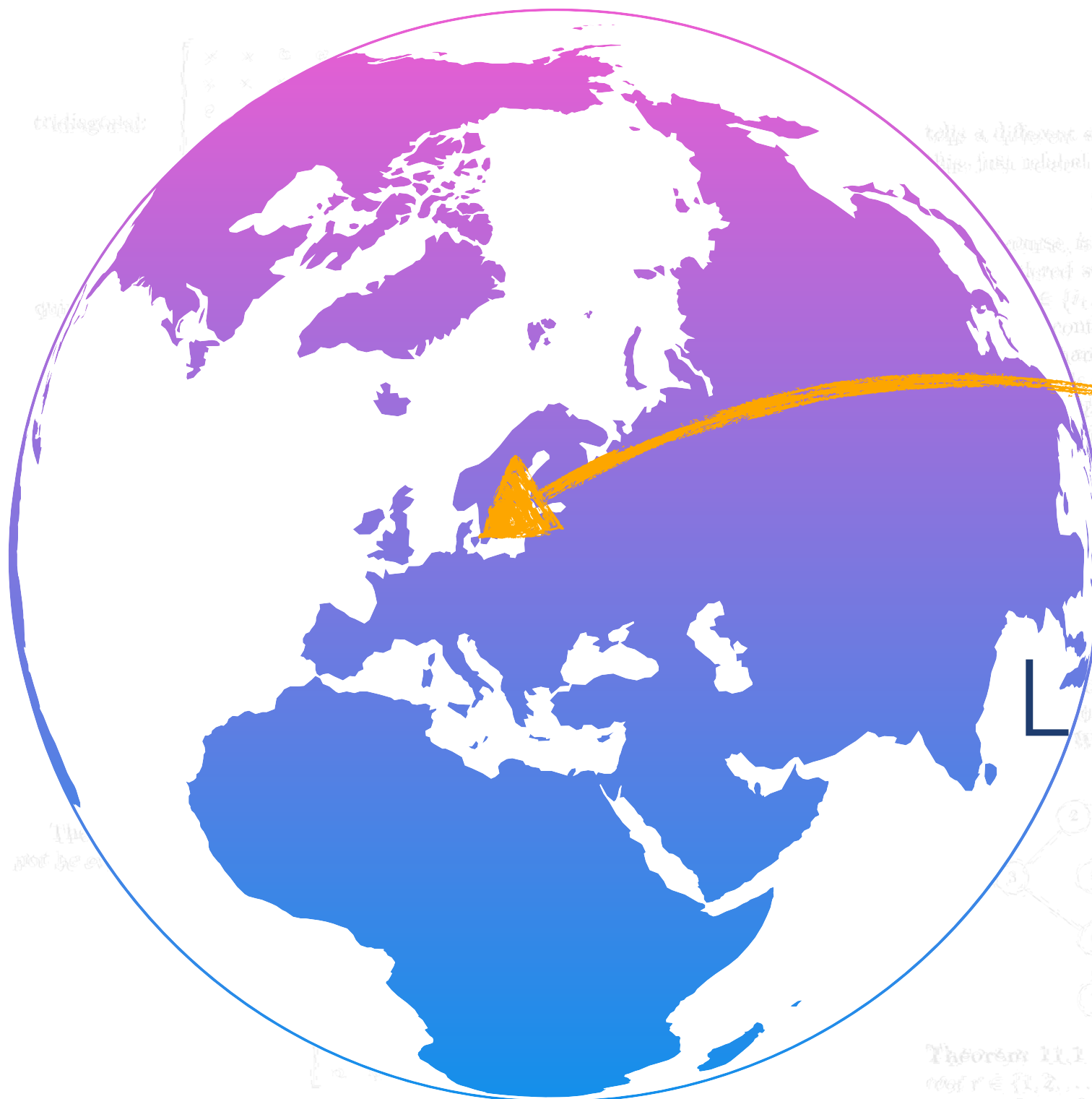
Every rooted tree can be monotonically ordered and, in general, such an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

of course, is equivalent to renumbering (renumbering counts!) the edges and vertices. The set of edges $\{(i, j) \in E : i < j\}$ is called a *path* joining the vertices i and j if $i, j \in \{1, 2, \dots, n\}$ and for every $s = 1, 2, \dots, n-1$ the set $\{(i, j) \in E : i < j \leq s\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a simple path. Both tridiagonal and arrowhead matrices correspond to trees. In the case of either quindriagonal or cyclic matrices when $n \geq 5$ and an arbitrary vertex $v \in V$ the path $T = (v, v)$ is called a *rooted tree* and v is the root. Unlike an ordinary graph, T admits a natural partial ordering that can best be explained by an analogy with a family tree. Thus, the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate this path, except for r and α , as a *predecessor* of α and a *successor* of r . The rooted tree T is *monotonically ordered* if each vertex is labelled monotonically; in other words, we label the vertices from the top of the tree to the bottom. (We have already said it, relabelling a graph is uninteresting in itself and the relabelling is the underlying point.) The tree can be seen monotonically ordered and, in general, such an ordering will give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} \times & \times & & & \\ & \times & \times & & \\ & & \times & \times & \\ & & & \times & \times \\ & & & & \times \end{bmatrix}$$

quintadiagonal:

$$\begin{bmatrix} \times & \times & & & & \\ & \times & \times & & & \\ & & \times & \times & & \\ & & & \times & \times & \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix}$$

The graph of A is not the same as the graph of L .

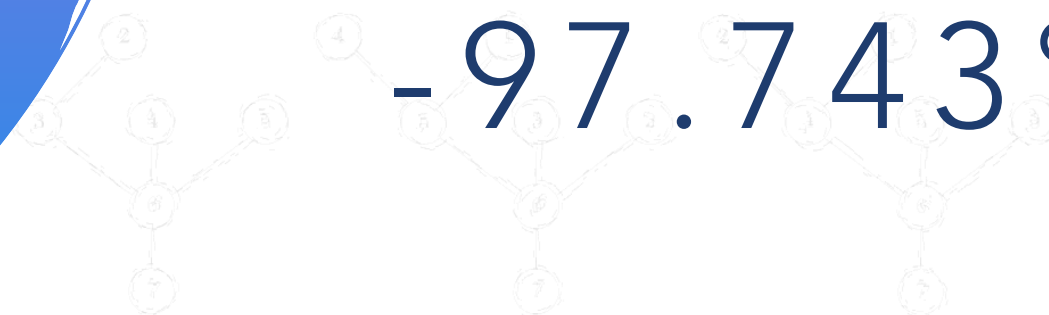
At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story - it is nothing other than the cycle matrix in disguise. To see this, just relabel the vertices as follows:

$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$

Of course, this relabelling (permuting round) the original undirected graph, produces a new set of edges $\{(i, j) \in E : (j, i) \in E\}$ called a *path* joining the vertices α and β if $\alpha \in \{i_0, i_1\}$, $\beta \in \{i_{n-1}, i_n\}$ and for every $s = 1, 2, \dots, n-1$ the set $\{i_s, i_{s+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a simple path. Both tridiagonal and quintadiagonal matrices correspond to trees. A graph with either quintadiagonal or cyclic matrices corresponds to a *rooted tree* T and an arbitrary vertex $r \in V$ is the root. The path $\{r, \alpha\}$ is called a *rooted path* and to be the root. Unlike an ordinary graph, T admits a natural partial ordering that can best be explained by an analogy with a family tree. Thus, the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate this path, except for r and α , as a *predecessor* of α and a *successor* of the root. The rooted tree T is *monotonically ordered* if each vertex is labelled monotonically; in other words, we label the vertices from the top of the tree to the bottom. (We have already said it, relabelling a graph is straightforward and the relabelling is the underlying matrix.) A tree can be ordered monotonically only and, in general, such an ordering does not exist. We may give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} x & & & & & \\ & x & & & & \\ 0 & x & x & & & \\ x & 0 & 0 & x & x & 0 \\ 0 & 0 & 0 & x & x & 0 \\ 0 & 0 & 0 & 0 & 0 & x \end{bmatrix}$$

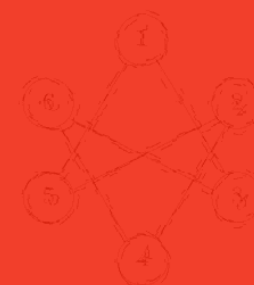
quintidiagonal:

symmetric:

cyclic:

The graph of A cannot be arbitrary.

Let us, instead, choose a graph G and think of the sparse matrix A which we have just displayed, but its graph,



RED

224

tells a different story - it is nothing other than, the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 3$

course, is equivalent to reordering (simultaneously) the equations and variables.

ordered set of edges $\{(i_k, j_k)\}_{k=1}^n \subseteq E$ is called a *path* joining the vertices a and b if $a \in \{i_1, j_1\}$, $b \in \{i_n, j_n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a simple path. Both tridiagonal and symmetric matrices correspond to trees.

is the case with either quintadiagonal or cyclic matrices when $n \geq 4$. For a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a *rooted tree* with r said to be the root. Unlike an ordinary tree, T admits a natural partial order, which can best be explained by an analogy with a family tree. Thus, r is the ancestor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, each $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that a rooted tree T is *monotonically ordered* if each vertex α is joined to all its predecessors in other words, we label the vertices from the top of T to the bottom. (We have already said it, relabelling a graph is tantamount to relabelling its rows and the columns of the underlying matrix.)

Let us now show that the monotonic ordering and, in general, still an ordering of the vertices of G give three monotonic orderings of the sparse rooted tree.

Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = [a_{k,j}]$ is a Cholesky factorization, it is true that

$$a_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

GREEN

78

Red

BLUE

57



Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:
$$\begin{bmatrix} \times & \times & & & & \\ & \times & \times & & & \\ & & \times & \times & & \\ & & & \times & \times & \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix}$$

quintadiagonal:
$$\begin{bmatrix} \times & \times & \times & & & \\ & \times & \times & \times & & \\ & & \times & \times & \times & \\ & & & \times & \times & \times \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix}$$

symmetric:
$$\begin{bmatrix} \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \end{bmatrix}$$

cyclic:
$$\begin{bmatrix} \times & \times & & & & \times \\ \times & \times & \times & & & \\ & \times & \times & \times & & \\ & & \times & \times & \times & \\ & & & \times & \times & \times \\ & & & & \times & \times \end{bmatrix}$$

BUILDING BLOCKS

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

OF TESTS
$$\begin{bmatrix} \times & \times & \times & \times & \times & \times \\ & \times & \times & \times & \times & \times \\ & & \times & \times & \times & \times \\ & & & \times & \times & \times \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix}$$

As a friend of mine, who is studying for a first or second year degree in mathematics, once told me, just displayed, but its graph,



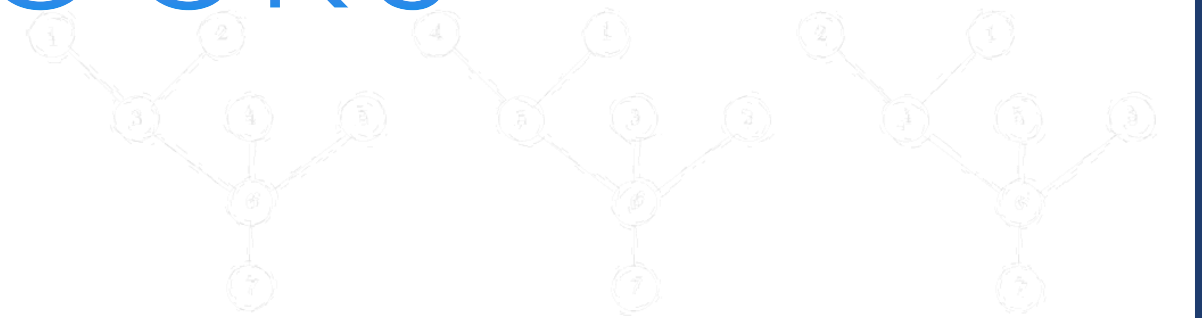
tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables. An ordered set of edges $\{(i_k, j_k)\}_{k=1}^v \subseteq E$ is called a *path* joining the vertices α and β if $\alpha \in \{i_1, j_1\}$, $\beta \in \{i_v, j_v\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and symmetric matrices correspond to trees, but this is not the case with either quintadiagonal or cyclic matrices when $p \geq 4$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (T, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

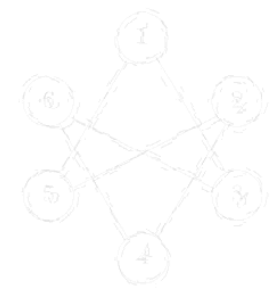
Every rooted tree can be monotonically ordered and, in general, still an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story - it is nothing other than the cycle matrix in disguise. To see this, just relabel the vertices as follows:

Create	Edit	Delete	Clone	Import
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☐	Name	Description
☒	Little Bobby Tables	Better not drop me!
☐	Big Bobby Tables	
☐	Foo Bar	Making up names is hard.

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is canonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} \times & \times & 0 & 0 & 0 & 0 \\ 0 & \times & \times & 0 & 0 & 0 \\ 0 & 0 & \times & \times & 0 & 0 \\ 0 & 0 & 0 & \times & \times & 0 \\ 0 & 0 & 0 & 0 & \times & \times \\ 0 & 0 & 0 & 0 & 0 & \times \end{bmatrix}$$


quintidiagonal:

$$\begin{bmatrix} \times & \times & 0 & 0 & 0 & 0 \\ \times & \times & \times & 0 & 0 & 0 \\ 0 & \times & \times & \times & 0 & 0 \\ 0 & 0 & \times & \times & \times & 0 \\ 0 & 0 & 0 & \times & \times & \times \\ 0 & 0 & 0 & 0 & \times & \times \end{bmatrix}$$


Page
In

Clone

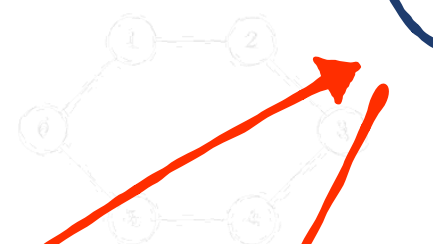
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Create

Page
Out



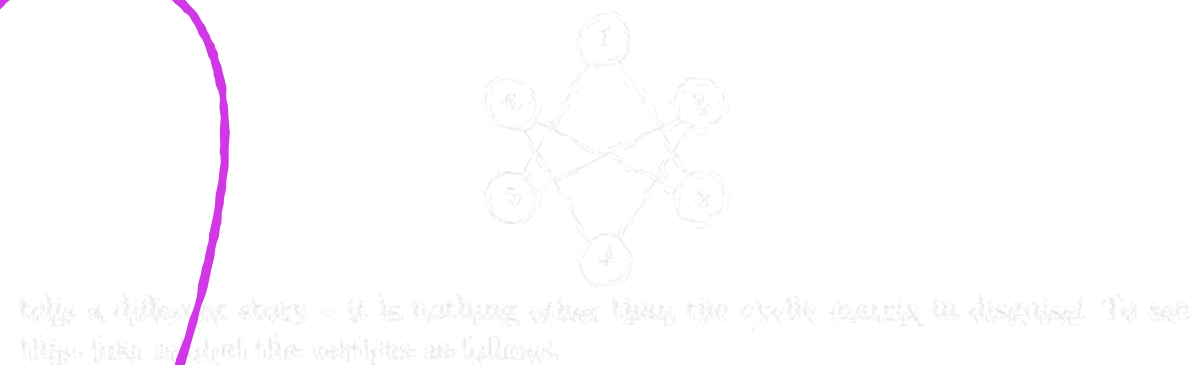
cyclic:

$$\begin{bmatrix} \times & \times & 0 & 0 & 0 & \times \\ \times & \times & \times & 0 & 0 & 0 \\ 0 & \times & \times & \times & 0 & 0 \\ 0 & 0 & \times & \times & \times & 0 \\ 0 & 0 & 0 & \times & \times & \times \\ \times & 0 & 0 & 0 & \times & \times \end{bmatrix}$$


The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & 0 & \times \\ \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & 0 & \times \\ \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & 0 & \times \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 3.$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_k, j_k)\}_{k=1}^v \subseteq E$ is called a *path* joining the vertices i and j if $i \in \{i_k, j_k\}$, $j \in \{i_k, j_k\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that T is a *tree* if all its members of V are joined by a unique simple path. Both undirected and directed trees are considered. But this is not the case with other quintidiagonal or cyclic matrices. Here,

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (T, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family structure. The vertex r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and the vertices are *successors* of r . More generally, if $u, v \in V$, u is said to be the *predecessor* of v and v is the *successor* of u if there is a path from u to v such that u is the first vertex along this path (except for u and v), as a predecessor of v and a successor of u . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to reordering the rows and the columns of the underlying matrix.)

Every rooted tree can be monotonically ordered and, in general, such an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{kj} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} x & x & 0 & 0 & 0 & 0 \\ 0 & x & x & 0 & 0 & 0 \\ 0 & 0 & x & x & x & x \\ 0 & 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & 0 & 0 & x \end{bmatrix}$$



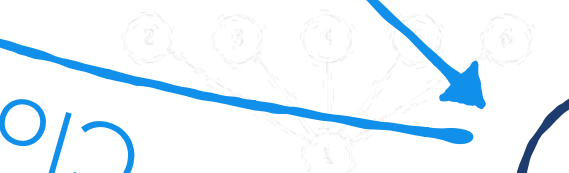
quintidiagonal:

$$\begin{bmatrix} x & x & 0 & 0 & 0 & 0 \\ x & x & x & 0 & 0 & 0 \\ 0 & x & x & x & x & 0 \\ 0 & 0 & x & x & x & x \\ 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & x & x \end{bmatrix}$$



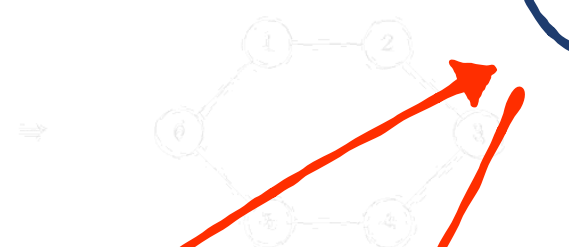
symmetric:

$$\begin{bmatrix} x & x & x & x & x & x \\ x & x & 0 & 0 & 0 & 0 \\ x & 0 & x & x & x & x \\ x & 0 & 0 & 0 & x & x \\ x & 0 & 0 & 0 & 0 & x \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} x & x & 0 & 0 & 0 & x \\ x & x & x & x & 0 & 0 \\ 0 & x & x & x & x & 0 \\ 0 & 0 & x & x & x & x \\ x & 0 & 0 & x & x & x \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} x & 0 & x & 0 & x & 0 \\ 0 & x & 0 & x & x & 0 \\ x & 0 & x & 0 & 0 & x \\ 0 & x & 0 & x & 0 & x \\ x & x & 0 & x & 0 \\ 0 & 0 & x & 0 & x \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

Import

Clone

Delete

Edit



tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

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Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (T, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path, and we designate a vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to reordering the rows and the columns of the underlying matrix.)

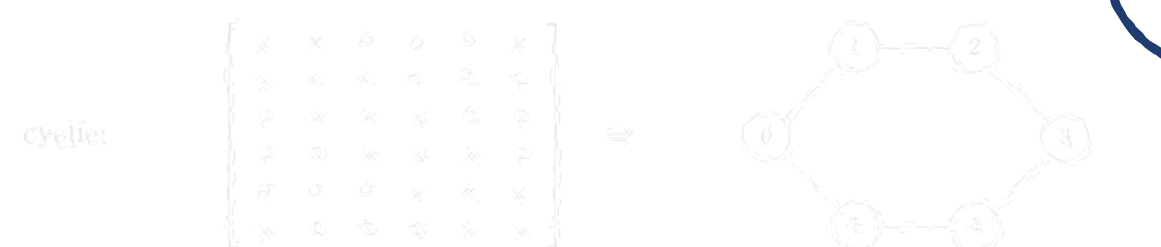
Every rooted tree can be monotonically ordered and, in general, still an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



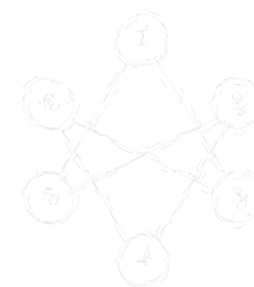
Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (T, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{kj} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



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tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

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Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (T, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family structure. If r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ has a unique *predecessor* β and an *arbitrary* vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of β . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

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The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & \times & 0 \\ \times & 0 & \times & 0 & 0 & \times \\ 0 & \times & 0 & \times & 0 & \times \\ \times & \times & 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times & 0 & \times \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

Therefore we will now give a list examples of matrices (represented by their sparsity patterns) and their graphs.

$(\mid (\&(a \geq b)(c \sim d))$
 $(\mid (A:dn:=B)(c=*)$
 $(!(\&(e \leq f)$
 $(:caseExact:=h)$
 $(\mid (i=j)(!(k \leq 1))))))$

just displayed, but its graph,

✓

$(\mid (\&(a \geq b)(c \sim d))(\mid (A:dn:=B)(c=*)(!(\&(e \leq f)(:caseExact:=h))))$

OR

+

FILTER

+

GROUP

AND

+

FILTER

+

GROUP

(a ≥ b)

(c ~ d)

OR

+

FILTER

+

GROUP

(A:dn:=B)

(c=*)

NOT

+

FILTER

+

GROUP

AND

+

FILTER

+

GROUP

(e ≤ f)

(:caseExact:=h)

OR

+

FILTER

+

GROUP

$$k_{ij} = \frac{a_{ki}}{b_{kj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their ranks.

(| (! (a >= b) (c ~ d))

(| (A:dn := B) (c = *)

(! (& (e <= f)

(: caseExact := h)

(| (i = j) (! (k <= 1))))))))

!

(

(

!

(

a

>

=

b

)

(

c

~

d

)

)

(

|

(

A

:

dn

:=

B

)

(

c

=

*

)

!

(

&

(

e

<

=

f

)

:

case

E

>

)

)

)

OR

+ FILTER

GROUP

NOT

+ FILTER

GROUP

You can negate only 1 filter in a group.

(a>=b)

(c~d)

You need to use ~=.

OR

+ FILTER

GROUP

(A:dn := B)

You need to trim the attribute and filter type.

(c=*)

NOT

+ FILTER

GROUP

AND

+ FILTER

GROUP

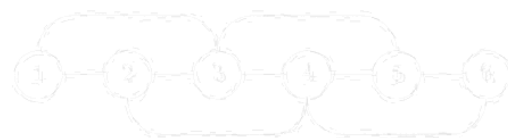
(e<=f)

(:caseExact:=h)

Therefore we will now give a few examples of matrices (represented by their sparsity patterns) and their graphs.

(id>=123)

SIMPLE



(email=*)

PRESENT

(cn=Jane*)

(:dn:1.2.3.4:=Jay)

SUBSTRING

EXTENSIBLE

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

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$$(\&(\quad)\dots(\quad))$$



$$(!(\quad))$$

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} x & 0 & x & 0 & x & 0 \\ 0 & x & 0 & x & x & 0 \\ x & 0 & x & 0 & 0 & x \\ 0 & x & 0 & x & 0 & x \\ x & x & 0 & 0 & x & 0 \\ 0 & 0 & x & x & 0 & x \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

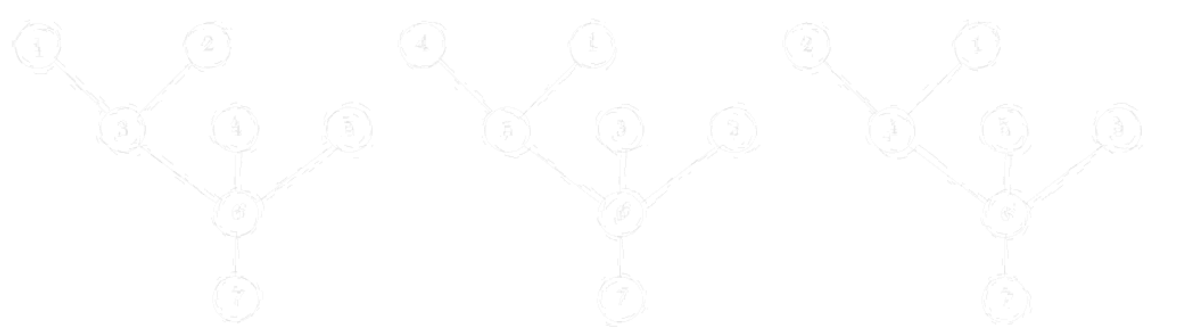
$$(1(\quad)\dots(\quad))$$

$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 3.$

This, of course, is equivalent to reordering (simultaneously) the equations and variables. An ordered set of edges $\{(i_k, j_k)\}_{k=1}^v \subseteq E$ is called a *path* joining the vertices i and j if $i = i_1, j = j_v, i_k \neq j_l$ and for every $k = 1, 2, \dots, v-1$ the set $\{(i_k, j_k) \cap \{i_l, j_l\} \mid l = 1, 2, \dots, v\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both triidiagonal and arrowhead matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 4$.

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Therefore we will now give a few examples of matrices (represented by their sparsity patterns) and their graphs.

```
function addOne(x) {
  if (Number.isFinite(x)) {
    x = x + 1;
  } else {
    console.log('error');
  }
  return x;
}
```

just displayed, but its graph,

tells a different story - it is nothing other than a cycle matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 6, \quad 5 \rightarrow 4, \quad 6 \rightarrow 3.$$

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An ordered set of edges $\{(i_k, j_k)\}_{k=1}^v \subseteq E$ is called a *path* joining the vertices i and j if $i \in \{i_k, j_k\}$, $j \in \{i_k, j_k\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tri-diagonal and arrowhead matrices correspond to trees, but this is not the case with either quindagonal or cyclic matrices when $p > 2$.

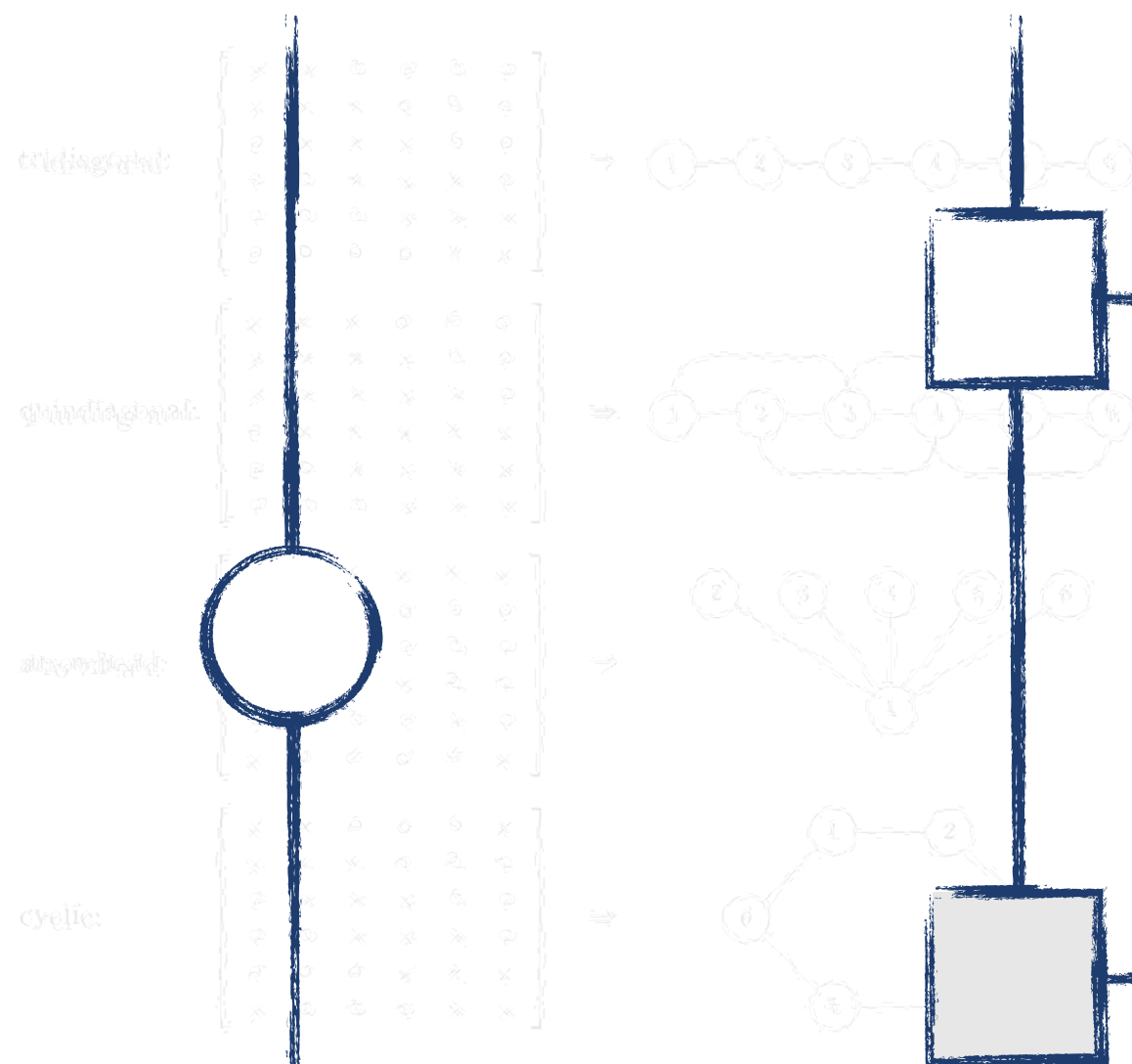
Given a tree G and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an undirected graph, T admits a natural ordering, which can best be explained by an analogy with a family tree. The root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labeled before all its predecessors; in other words, if the vertices grow the top of the tree to the root. (As we have already said, this is a simple but important tool to permuting the rows and the columns of the matrix.)

Every rooted tree can be monotonically ordered, in general, still an ordering is not unique. We now give three monotonic orderings of the same rooted tree.

Theorem 11 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$. Assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, that

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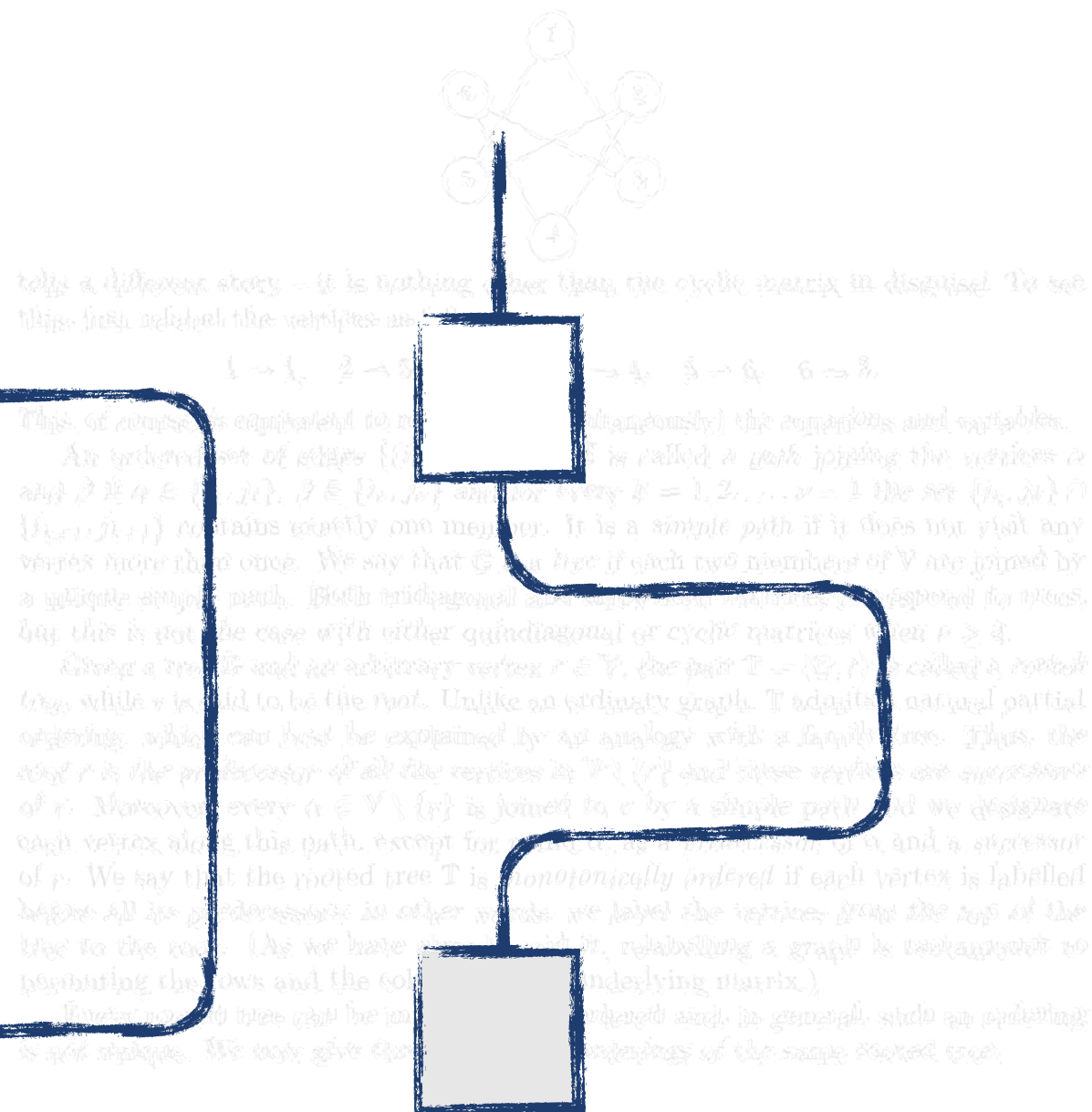


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$$\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & \times & 0 \\ \times & 0 & \times & 0 & 0 & \times \\ 0 & \times & 0 & \times & 0 & \times \\ \times & 0 & 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times & 0 & \times \end{bmatrix}$$

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IF



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tridiagonal:

$$\begin{bmatrix} x & x & 0 & 0 & 0 & 0 \\ 0 & x & x & 0 & 0 & 0 \\ 0 & 0 & x & x & 0 & 0 \\ 0 & 0 & 0 & x & x & 0 \\ 0 & 0 & 0 & 0 & x & x \end{bmatrix}$$


quintidiagonal:

$$\begin{bmatrix} x & x & x & x & x & 0 \\ 0 & x & x & x & x & x \\ 0 & 0 & x & x & x & x \\ 0 & 0 & 0 & x & x & x \end{bmatrix}$$


symmetric:

$$\begin{bmatrix} x & x & x & x & x & x \\ x & 0 & 0 & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$


cyclic:

$$\begin{bmatrix} x & x & 0 & 0 & 0 & 0 \\ x & x & x & 0 & 0 & 0 \\ 0 & x & x & 0 & 0 & 0 \\ 0 & 0 & x & x & 0 & 0 \\ 0 & 0 & 0 & x & x & 0 \\ x & 0 & 0 & 0 & 0 & x \end{bmatrix}$$


The graph of a matrix often reveals at a single glance its structure, which may not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} x & 0 & x & 0 & x \\ 0 & x & 0 & x & x \\ x & 0 & x & 0 & 0 \\ 0 & x & 0 & x & 0 \\ x & x & 0 & 0 & x \\ 0 & 0 & x & x & 0 \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices we have just displayed, but its graph,

if the four matrices we have



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$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

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Therefore we will now give a few examples of matrices (represented by their sparsity

1

```
// TODO: Write tests later  
test('it renders', async function(assert) {  
  await render(hbs`<ComplexComponent />`);  
  assert.ok(true);  
});
```

just displayed, but its graph,

$$t_{k,j} = \frac{a_{k,j}}{t_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (1.5)$$

```
import { percySnapshot } from 'ember-percy';  
  
...  
  
// TODO: Write tests later  
  
test('complex workflow', async function(assert) {  
  await visit('/complex-page');  
  await percySnapshot(assert);  
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} \times & \times & & & & \\ & \times & \times & & & \\ & & \times & \times & & \\ & & & \times & \times & \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix}$$


quintidiagonal:

$$\begin{bmatrix} \times & \times & \times & \times & \times & \\ & \times & \times & \times & \times & \\ & & \times & \times & \times & \\ & & & \times & \times & \\ & & & & \times & \times \\ & & & & & \times \end{bmatrix}$$


symplectic:

$$\begin{bmatrix} \times & & & \times & & \\ & \times & & & \times & \\ & & \times & & & \times \\ & & & \times & & \\ & & & & \times & \\ & & & & & \times \end{bmatrix}$$


cyclic:

$$\begin{bmatrix} \times & & & & \times & \\ & \times & & & & \\ & & \times & & & \\ & & & \times & & \\ & & & & \times & \\ & & & & & \times \end{bmatrix}$$


THE GRAPH OF A MATRIX OFTEN REVEALS AT A GLANCE ITS STRUCTURE, WHICH MIGHT NOT BE EVIDENT FROM ITS SPARSITY PATTERN. THIS CAN BE SEEN IN THE MATRIX

$$\begin{bmatrix} \times & & \times & & \times & & \\ & \times & & \times & & \times & \\ \times & & \times & & \times & & \\ & \times & & \times & & \times & \\ \times & & \times & & \times & & \\ & \times & & \times & & \times & \\ \times & & \times & & \times & & \end{bmatrix}$$

It is clear that such a matrix is symmetric (as indicated by the symmetry of the sparsity pattern) and we know just displayed, but its graph,



tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

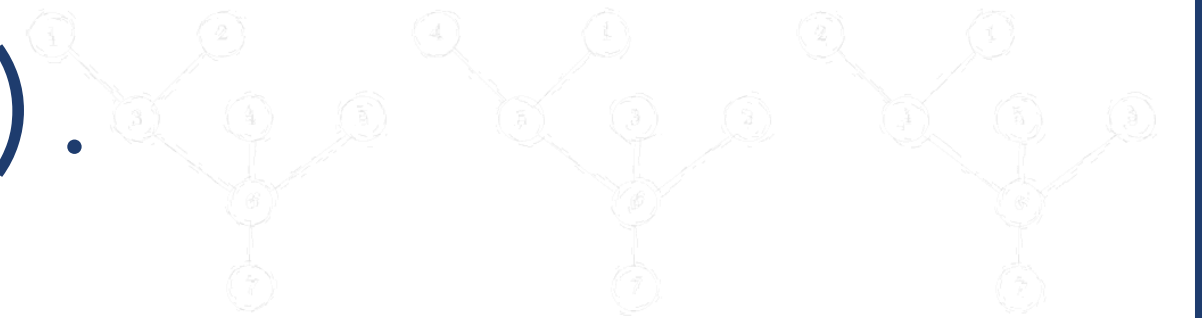


Of course, it is equally so concerning simultaneously the equations and variables.

A graph with vertices $\{1, 2, \dots, v\}$ and edges $\{(i, j)\} \subseteq E$ is called a *path* joining the vertices α and β if $\alpha \in \{i_1, i_2\}$, $\beta \in \{i_v, i_v\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, i_{k+1}\} \cap \{i_{k+1}, i_{k+2}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and arrowhead matrices correspond to trees, but this is not the case with either quintidiagonal or cyclic matrices when $v \geq 4$.

A tree $T = (G, r)$ is called a *rooted tree* if $r \in V$, the node r is called a *root* (note that this is not to be confused with the root of a polynomial). There is a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors (in other words, we label the vertices from the top of the tree to the bottom, i.e. we label exactly said, labelling a graph is tantamount to permutation of the rows and the columns of the underlying matrix.)

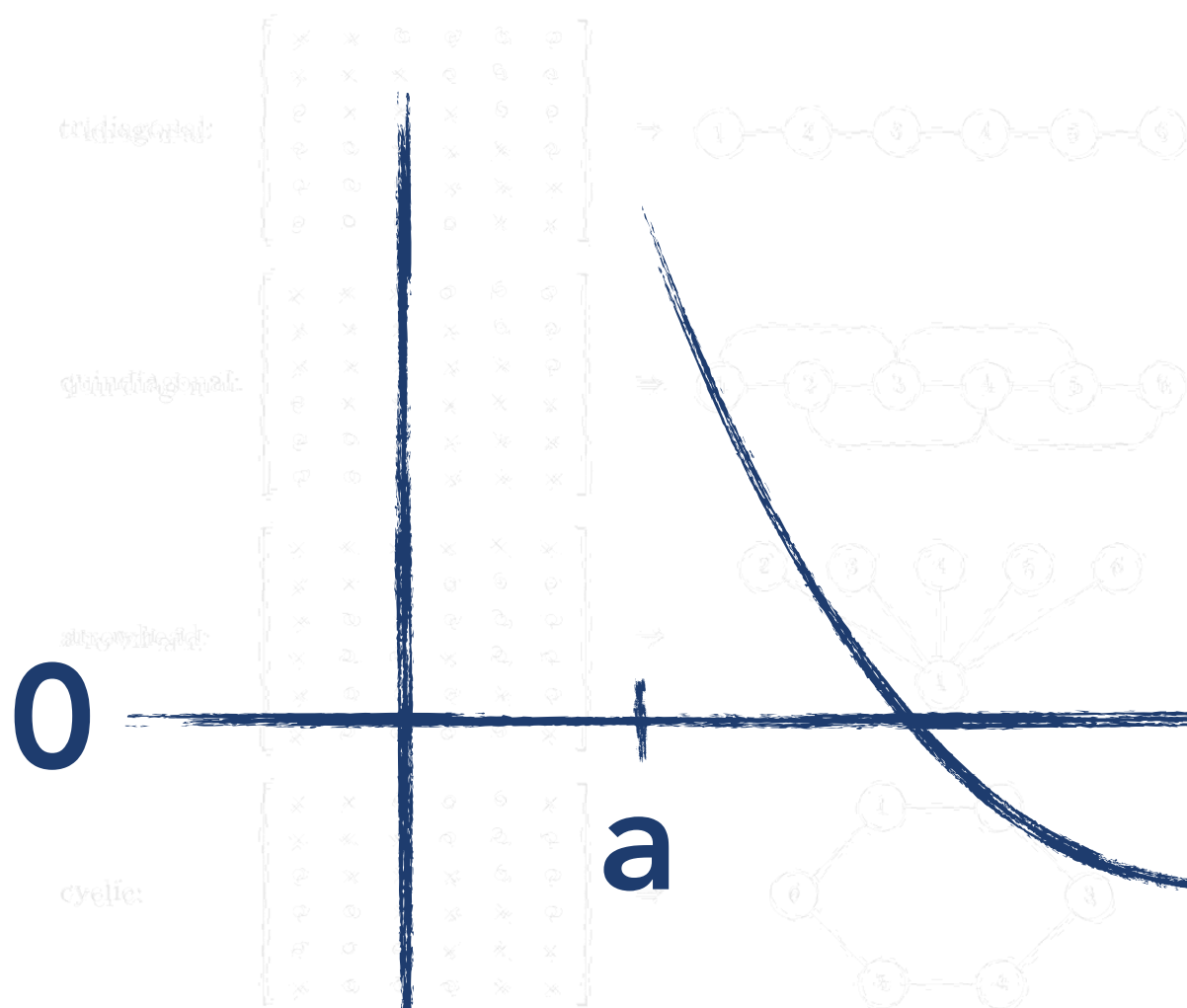
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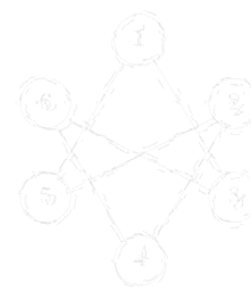
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$$\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & \times & 0 \\ \times & 0 & \times & 0 & 0 & \times \\ 0 & \times & 0 & \times & 0 & \times \\ \times & \times & 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times & 0 & \times \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_k, j_k)\}_{k=1}^v \subseteq E$ is called a *path* joining the vertices α and β if $\alpha \in \{i_1, j_1\}$, $\beta \in \{i_v, j_v\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and arrowhead matrices correspond to trees, but this is not the case with either quindagonal or cyclic matrices when $p \geq 4$.

Given a tree G and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, if $\alpha \in V \setminus \{r\}$ is the father of β , then β is the *son* of α and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors. In other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree can be monotonically ordered and, in general, such an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity

pattern) and their graphs.

YOU CAN FIND

EQUALLY MANY NUMBERS

BETWEEN 0 AND 1

AS YOU CAN

BETWEEN $-\infty$ AND ∞ .

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

As a kind of exercise, let us try to establish the link between the two quantities which are here just displayed, but its graph,



tell a different story - it is nothing other than the cyclic matrix in disguise. To see this, let us relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_k, j_k)\}_{k=1}^n \subseteq E$ is called a *path* joining the vertices α and β if $\alpha \in \{i_1, j_1\}$, $\beta \in \{i_n, j_n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it contains no vertex more than once. It is said that G has a *path* if such two numbers α, β are joined by a (unique) simple path. Both triangulated and acyclic graphs are *path-connected*, but this is not the case with either quindagonal or cyclic matrices when $p \geq 4$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (T, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path, and we designate each vertex α on this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

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Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

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Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6$$

$$\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & \times & 0 \\ \times & 0 & \times & 0 & 0 & \times \\ 0 & \times & 0 & \times & 0 & \times \\ \times & \times & 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times & 0 & \times \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

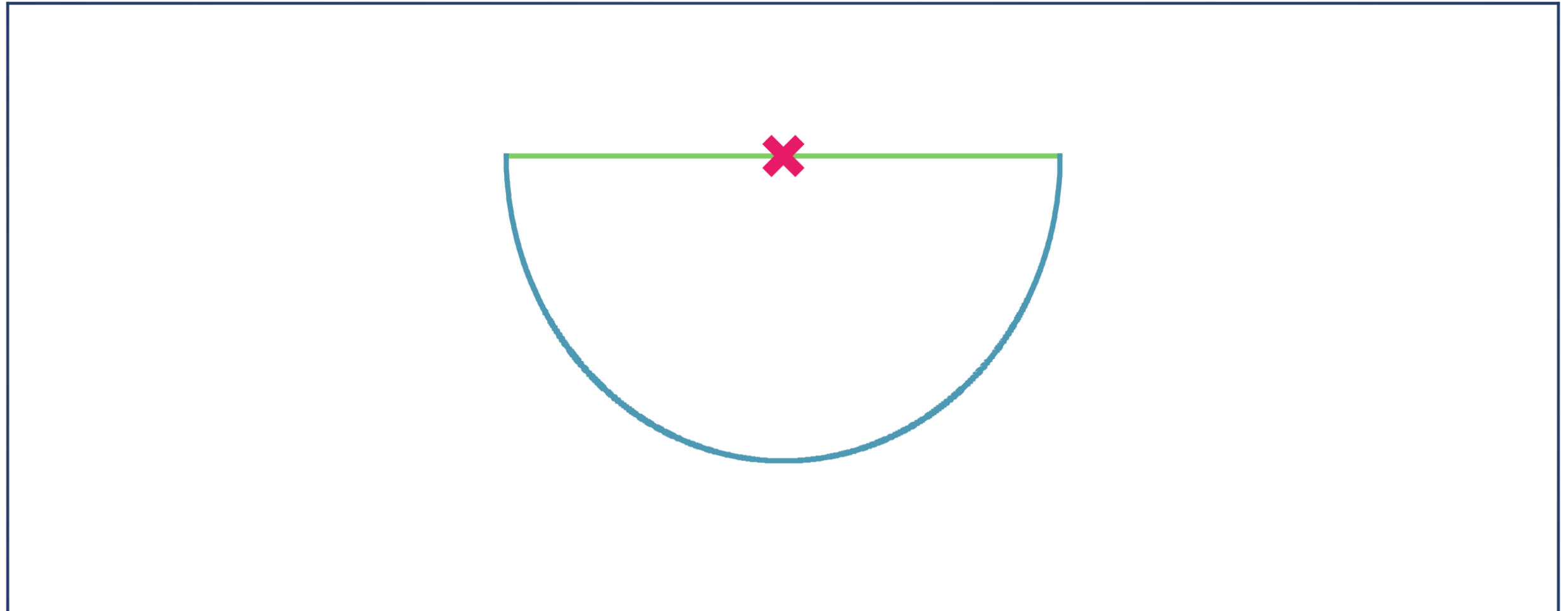
$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



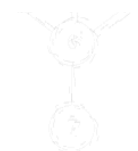
tells a different story - it is nothing other than the cycle matrix in disguise. To see this, just relabel the vertices as follows:

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{pmatrix} \rightarrow \begin{pmatrix} 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}$$



$$\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & \times & 0 \\ \times & 0 & \times & 0 & 0 & \times \\ 0 & \times & 0 & \times & 0 & \times \\ \times & \times & 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times & 0 & \times \end{bmatrix}$$

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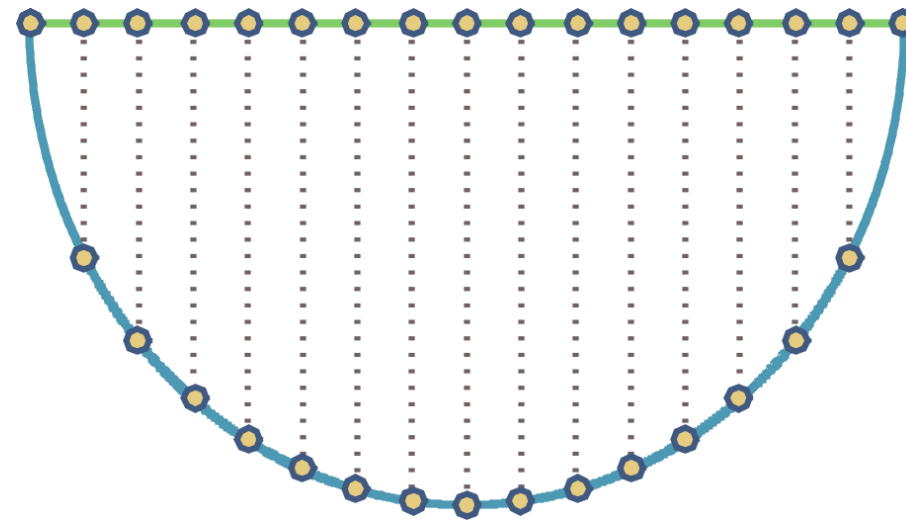
$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



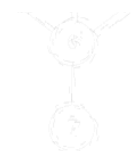
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$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{pmatrix} \rightarrow \begin{pmatrix} 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}$$



$$\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & \times & 0 \\ \times & 0 & \times & 0 & 0 & \times \\ 0 & \times & 0 & \times & 0 & \times \\ \times & \times & 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times & 0 & \times \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are deep arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

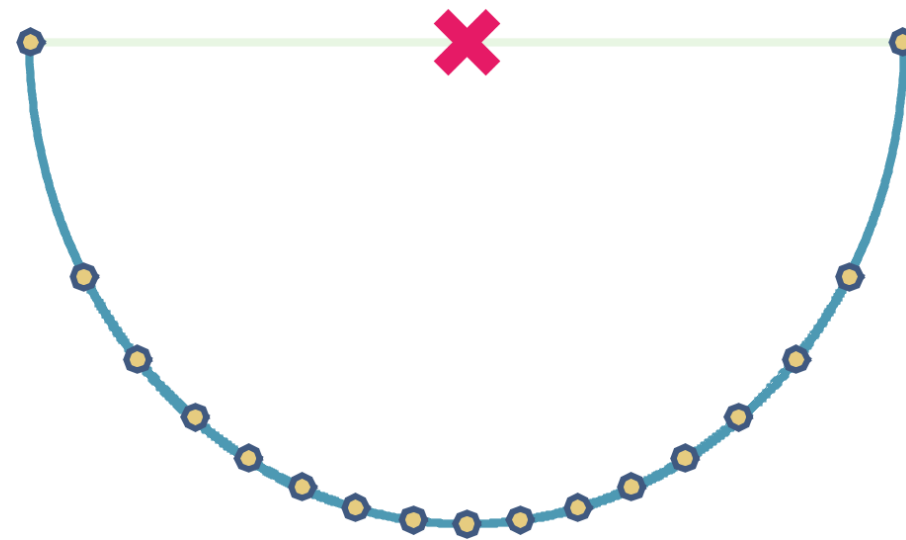
$$l_{kj} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

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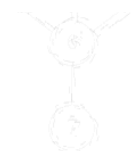


tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{pmatrix} \rightarrow \begin{pmatrix} 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}$$



$$\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & \times & 0 \\ \times & 0 & \times & 0 & 0 & \times \\ 0 & \times & 0 & \times & 0 & \times \\ \times & \times & 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times & 0 & \times \end{bmatrix}$$



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A have been permuted so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

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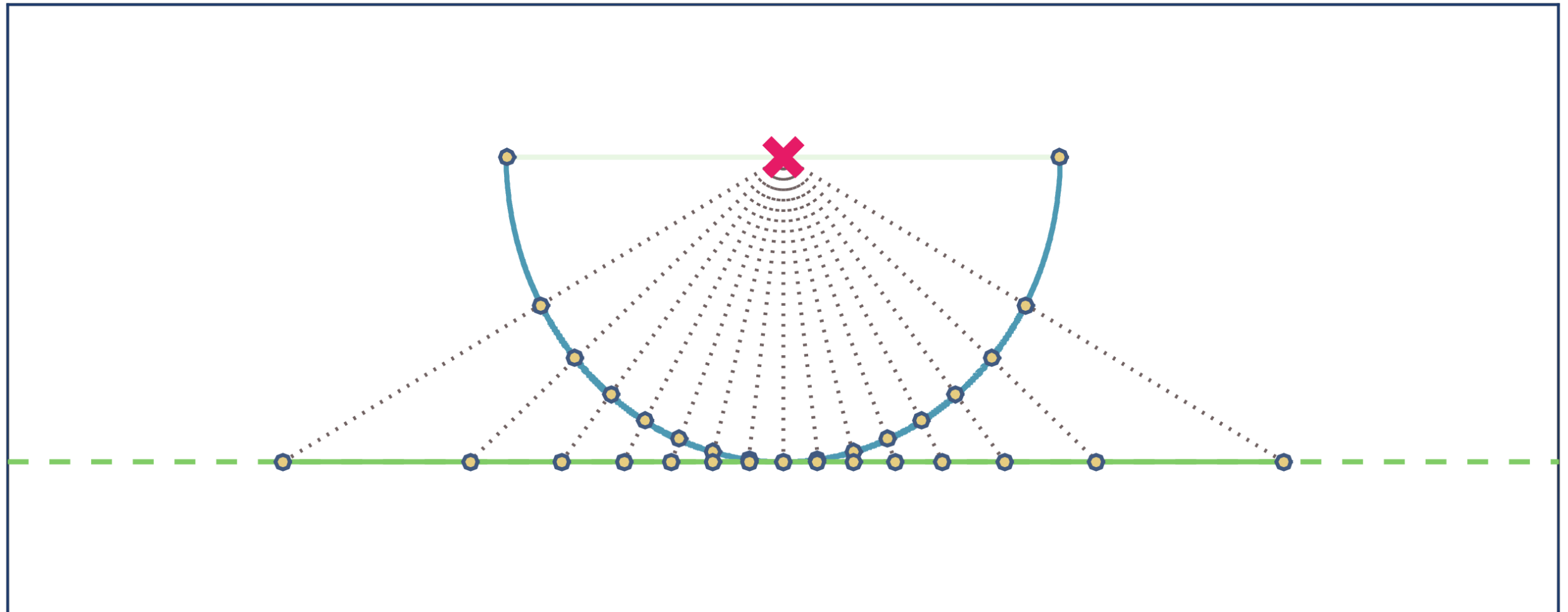
At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



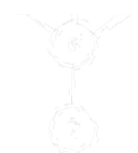
tells a different story - it is nothing other than the cyclic matrix in disguise. To see this, just relabel the vertices as follows:

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{pmatrix} \rightarrow \begin{pmatrix} 3 \\ 4 \\ 5 \\ 6 \\ 1 \\ 2 \end{pmatrix}$$



$$\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & \times & 0 \\ \times & 0 & \times & 0 & 0 & \times \\ 0 & \times & 0 & \times & 0 & \times \\ \times & \times & 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times & 0 & \times \end{bmatrix}$$

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Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



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$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{pmatrix} \rightarrow \begin{pmatrix} 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & \times & 0 \\ \times & 0 & \times & 0 & 0 & \times \\ 0 & \times & 0 & \times & 0 & \times \\ \times & \times & 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times & 0 & \times \end{bmatrix}$$

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1

5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

Running:

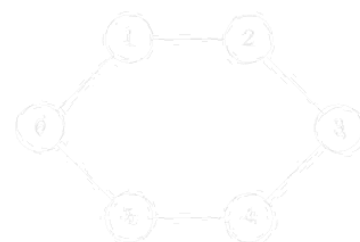
acyclic:

$$\begin{bmatrix} \times & \circ & \times & \circ & \times & \circ \\ \times & \times & \times & \times & \times & \times \\ \times & \circ & \circ & \circ & \circ & \times \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} \times & \times & \circ & \circ & \circ & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} \times & \circ & \times & \circ & \times & \circ \\ \circ & \times & \circ & \times & \times & \circ \\ \times & \circ & \times & \circ & \circ & \times \\ \circ & \times & \circ & \times & \circ & \times \\ \times & \times & \circ & \circ & \times & \times \\ \circ & \circ & \times & \times & \circ & \times \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

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1

5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



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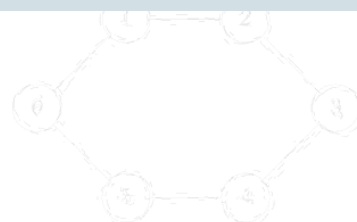
1 assertion of 1 passed, 0 failed.

If, if, if

cyclic:



$\Rightarrow$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

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Therefore we will now give a few examples of matrices (represented by their sparsity

5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



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2 assertions of 2 passed, 0 failed.

If, if, if

Use common, everyday words

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} \times & \emptyset & \times & \emptyset & \times & \emptyset \\ \emptyset & \times & \emptyset & \times & \times & \emptyset \\ \times & \emptyset & \times & \emptyset & \emptyset & \times \\ \emptyset & \times & \emptyset & \times & \emptyset & \times \\ \times & \times & \emptyset & \emptyset & \times & \emptyset \\ \emptyset & \emptyset & \times & \times & \emptyset & \times \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



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5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

3 assertions of 3 passed, 0 failed.

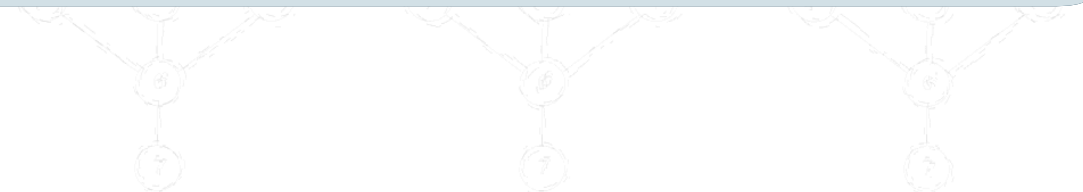
If, if, if

Use common, everyday words

Write less with theorems and new terms

$$\begin{bmatrix} \times & \circ & \times & \circ & \times & \circ \\ \circ & \times & \circ & \times & \times & \circ \\ \times & \circ & \times & \circ & \circ & \times \\ \circ & \times & \circ & \times & \circ & \times \\ \times & \times & \circ & \circ & \times & \circ \\ \circ & \circ & \times & \times & \circ & \times \end{bmatrix}$$

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5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

4 assertions of 4 passed, 0 failed.

If, if, if

Use common, everyday words

Write less with theorems and new terms

All your basis are belong to us

$$\begin{pmatrix} \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \end{pmatrix}$$

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5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline ▼

EmberFest 18.10.2019

5 assertions of 5 passed, 0 failed.

If, if, if

Use common, everyday words

Write less with theorems and new terms

All your basis are belong to us

1 picture = 1000 words

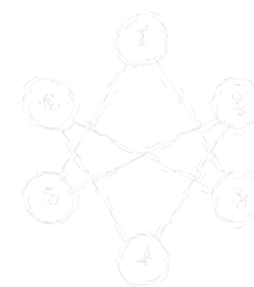
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ 0 & \times & 0 & \times & \times & 0 \\ \times & 0 & \times & 0 & 0 & \times \\ 0 & \times & 0 & \times & 0 & \times \\ \times & \times & 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times & 0 & \times \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story - it is nothing other than the cycle matrix in disguise. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 4, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

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Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (T, r)$ is called a *rooted tree*, where r is said to be the *root*. Unlike an ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree can be monotonically ordered and, in general, such an ordering is not unique. We now give three monotonic orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are taken in such a way that $T = (G, r)$ is monotonically ordered. Given that $x = [x_i]_{i=1}^d$ is a *boundary factorization*, it is true that

$$t_{k,j} = \frac{a_{k,j}}{t_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Q.E.D.